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Coupling Functions for Muon Hodoscopes

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Abstract—The specific features of calculating the coupling functions for muon hodoscopes are considered. The results of calculating the main coupling functions for three versions of the primary spectra of cosmic rays (CR)—galactic CR, solar CR during proton events, and galactic CR variations during Forbush decreases—are presented. The calculations have been performed based on modeling CR propagation through the Earth's atmosphere using the CORSIKA code. The mean and median CR energies, contributing to the muon detector counting rate, have been calculated for different zenith angles. It has been shown that muon detectors are sensitive to different energies of primary CRs for different directions.

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INTRODUCTION

An analysis of CR variations is one of the methods for studying solar, heliospheric, and geomagnetic processes. Data on the primary CR flux and information from various ground-based detectors can be studied in this case. Neutron monitors (NMs), muon telescopes, and different muon detectors of setups used to register extensive air showers (EASs), are usually used to register ground-level CRs. Muon hodoscopes, which make it possible to simultaneously measure spatial–angular characteristics of a CR muon flux from all directions, open new possibilities for CR variation ground-based monitoring [1]. When analyzing primary CR flux variations with the help of surface detectors, one should use the coupling functions that make it possible to compare registered counting rate variations and CR intensity changes at the boundary of the atmosphere [2]. The coupling functions were previously calculated mainly for NMs (see the review [3]) and muon detectors of setups used to register EASs [4, 5].

The present work considers the specific features of the coupling function calculation for muon hodoscopes and coordinate detectors registering the differential muon flux with respect to angles. The specific calculations were performed for the DECOR coordinate detector [6] and for the URAGAN muon hodoscope [7] of the NEVOD experimental complex (MEPhI).

COUPLING FUNCTION CALCULATION METHOD

Coupling functions determine relations between spectra of primary particles and those of particles on the Earth's surface. A ground-level (GL) muon flux, differential with respect to energy, can be written as follows:

$$J_\mu^{GL}(E_\mu, \theta) = \int_{E_{\min}}^{\infty} m^{GL}(E_\mu, E, \theta) J_p(E) dE, \quad (1)$$

where $m^{GL}(E_\mu, E, \theta)$ is the total muon energy (E_μ) distribution function at a certain zenith angle θ on the Earth's surface for a primary proton with energy E, $J_p(E)$ is the differential intensity of a primary proton flux. It is assumed that muon retains the proton direction of motion. In this case the setup counting rate is determined by the expression

$$\begin{aligned} n &= \int_{E_{\text{thr}}}^{\infty} dE_\mu \int d\Omega J_\mu(E_\mu, \theta) S(\theta, \varphi) \\ &= \int_{E_{\min}}^{\infty} dE \int d\Omega S(\theta, \varphi) J_p(E) M(E, \theta), \end{aligned} \quad (2)$$

where E_{thr} is the threshold energy of registered particles, $S(\theta, \varphi)$ is the setup effective area for this direction of muon arrival, $E_{\min}$ is the primary proton threshold energy mainly dependent on the cutoff rigidity, and $M(E, \theta)$ is the multiplicity function (the mean number of muons with energies higher than the threshold energy (E_{thr}) for one proton with energy E on the Earth's surface at a certain zenith angle θ)

$$M(E, \theta) = \int_{E_{\text{thr}}}^{E} m^{GL}(E_\mu, E, \theta) dE_\mu. \quad (3)$$

The muon hodoscope makes it possible to measure a muon flux independently for each direction of the

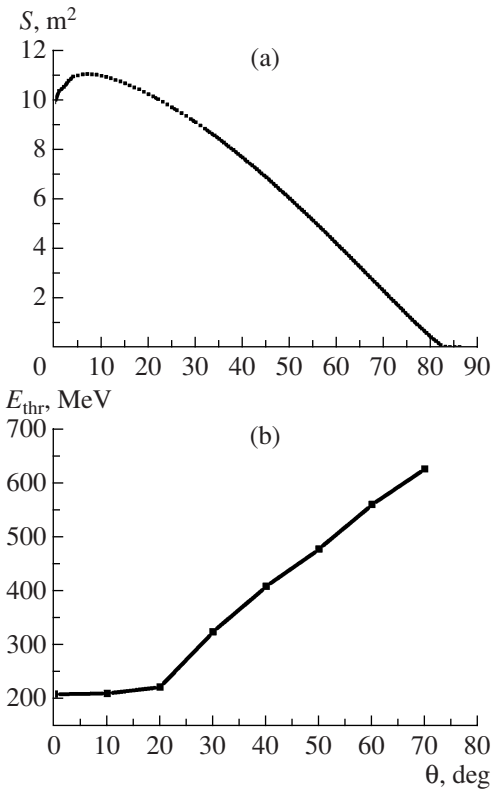

Fig. 1. The parameters of the URAGAN muon hodoscope depending on the zenith angle: (a) the effective area of one URAGAN SM, and (b) the mean total threshold energy of muon.

celestial hemisphere; therefore, it is convenient to define the collection function using the formula

$$P(E, \theta_i, \varphi_j) = \int_{\Delta\Omega} d\Omega S(\theta, \varphi) M(E, \theta), \quad (4)$$

where $\Delta\Omega$ is the interval of the spatial angle around the direction (θ_i, φ_j).

The yield function relates the hodoscope differential (with respect to the primary proton energy) counting rate in the direction (θ_i, φ_j) to the primary proton flux:

$$\frac{dn(\theta_i, \varphi_j)}{dE} = P(E, \theta_i, \varphi_j) J_p(E). \quad (5)$$

The expression in the right-hand side of equality (5) is the hodoscope response function in this direction $G(E, \theta_i, \varphi_j)$. Such a definition makes it also possible to study the cases when the flux of primary CRs is anisotropic (in this case $J_p(E)$ can depend on the angle). If the $M(E, \theta)$ function slightly varies within the spatial angle $\Delta\Omega$, expression (4) can be written as

$$P(E, \theta_i, \varphi_j) = M(E, \theta_i) \Delta S\Omega(\theta_i, \varphi_j), \quad (6)$$

where $\Delta S\Omega(\theta_i, \varphi_j)$ is the setup partial aperture near the direction (θ_i, φ_j).

Thus, the functions of multiplicity $M(E, \theta)$, yield $P(E, \theta)$, and response $G(E, \theta)$ are the main functions that should be calculated in order to study CR variations. In this work these functions were calculated by modeling the passage of primary protons with energies of 1 GeV to 10 TeV through the atmosphere at zenith angles of 0°–70° with the help of the CORSIKA v6. 611 code [8]. This work uses the SIBYLL-2.1 (for a hadron energy of $E_h > 80$ GeV) and FLUKA 2006 (at lower energies) combination of models to model hadron interactions.

DESCRIPTION OF DETECTORS

The coupling functions were calculated for the DECOR coordinate detector and the URAGAN muon hodoscope. The DECOR detector has a modular structure and consists of ten supermodules (SMs) surrounding the NEVOD water tank. The counting rate of the DECOR special trigger, which detects events where the upper and side supermodels operate, was used to study CR variations. The following parameters of the mean quantities were used to calculate the coupling functions under these trigger conditions for the DECOR detector: a threshold energy of 2 GeV, an aperture of 1.5 m^2 sr, and a muon arrival zenith angle of 46°.

The URAGAN muon hodoscope consists of three independent mobile assemblies with an area of 11.5 m^2 each. The dependence of the effective area on the zenith angle for one URAGAN SM is presented in Fig. 1a, taking into account the SM internal structure. Since the URAGAN muon hodoscope is located in the building of the NEVOD experimental complex (i.e., being surrounded by walls and roof), this hodoscope has different threshold energies of registered muons for different directions (Fig. 1b). The muon loss is composed of the loss in an SM matter, brick wall of the building (20°–70°) and/or concrete ceiling (0°–20°), and clayite layer over a ceiling. The path–energy tables from [9] were used to calculate the threshold energies.

RESULTS OF COUPLING FUNCTION MODELING

The simulation was performed at fixed energies of primary protons uniformly distributed over the energy logarithm at intervals of 0.1 (for energies lower than 20 GeV for 1 million of events) or 0.2 (for energies higher than 20 GeV for 100 thousands of events). According to the zenith angle, the simulation was performed at an interval of 10 deg. Figure 2 shows the results of the multiplicity function calculations for the URAGAN setup and the DECOR detector. The yield functions can be calculated using formula (6), where the values of the muon detector aperture are used (Table 1). A ring with the zenith angle boundaries in the middle of

Table 1. Aperture for different zenith angle ranges of the URAGAN setup and the DECOR detector

	URAGAN								DECOR
θ, deg	0	10	20	30	40	50	60	70	46
Interval θ, deg	0–5	5–15	15–25	25–35	35–45	45–55	55–65	65–75	–
$\Delta S\Omega$, $m^2 \cdot sr$	0.76	6.1	11.2	14.6	15.8	14.7	11.6	6.8	1.5

Table 2. The mean and median energies of primary protons at different zenith angles for the URAGAN and DECOR detectors

Spectrum / θ	Mean energies, GeV									Median energies, GeV								
	URAGAN								DECOR	URAGAN								DECOR
	0°	10°	20°	30°	40°	50°	60°	70°	46°	0°	10°	20°	30°	40°	50°	60°	70°	46°
GCRs	63	64	67	74	84	100	132	195	141	38	39	40	43	50	59	77	120	82
SCRs	12.5	12.7	13.3	14.5	16.5	20	26	38	–	7	8	9	10	11	13	17	26	–
FD	16	17	18	19	22	25	31	44	31	13	14	15	16	17	20	25	36	28

the range between the modeling grid nodes was taken as a spatial angle interval in order to calculate the aperture for the URAGAN muon detector, and the threefold effective area of one SM was used since URAGAN includes three SMs.

The following CR spectra (the energy is given in GeV) were used to calculate the response functions:

(1) the GCR spectrum [10]: $J_p(E) = 1.8E^{-2.7}$ [1/(cm^2 s sr GeV)],

(2) the SCR spectrum (the parameter values were obtained based on [11]): $J_p(E) = 2.3 \times 10^4 E^{-5.15}$ [1/(cm^2 s sr GeV)],

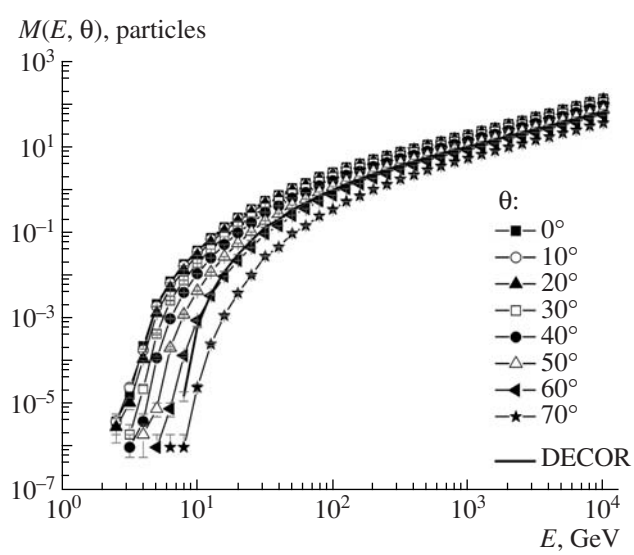

Fig. 2. The multiplicity functions for the URAGAN and DECOR setups (only the statistical errors are indicated).

(3) the GCR spectrum modulation for FD [12]: $\Delta J_p/J_p = 0.2R^{-1}$, where R is rigidity (GV).

Considering the response function as the distribution of detector responses over the primary CR energy, we can calculate the mean energy of primary protons (E_{mean}) that give the main contribution to the counting rate. To analyze the data of different setups, it is also convenient to use the median energy ($E_{0.5}$), which divides the detector response energy distribution into two equal parts according to the energy of primary CRs. Table 2 presents the mean and median energies of primary protons for different spectra. The dependence of the mean energies on the zenith angle for GCRs is adequately described by the simple formula $E_{mean}(\theta) = 63\cos^{-1.08}(\theta)$ GeV.

CONCLUSIONS

We considered the specific features of the calculation of the coupling functions for the muon hodoscopes and coordinate detectors. It was shown that muon hodoscopes in different directions are sensitive to primary protons with different energies. Based on the simulation using the CORSIKA code, the functions of multiplicity, yield, and response of the URAGAN and DECOR detectors and the mean and median energies of primary protons for different versions of the primary spectrum were calculated. The results make it possible to estimate different variations in the primary CR flux in the region of primary spectrum energies of about 10 GeV and higher, to which muon detectors are sensitive, based on the muon counting rate variations.

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