The coupling between primary and secondary cosmic-ray muon variations at high energies

1971 J. Phys. A: Gen. Phys. 4 660

(http://iopscience.iop.org/0022-3689/4/5/008)

J. Phys. A: Gen. Phys., 1971, Vol. 4. Printed in Great Britain

The coupling between primary and secondary cosmic-ray muon variations at high energies

F. EL BEDEWI and A. GONED

Physics Department, Faculty of Science, Ein Shams University, Cairo, UAR

MS. received 26*th March* 1971

Abstract. The muon sea-level integral multiplicity and the coefficients of coupling between primary and secondary muon variations were derived as functions of primary energy for various absorber thicknesses and zenith angles using the Cocconi–Koester–Perkins interaction model. The calculated multiplicities were found to agree at low energies with those derived from geomagnetic data and to be in general accordance with those obtained from nucleon cascade calculations at high energies. The calculated coupling coefficients were observed to depend weakly on zenith angle and to change with primary energy E as E^3 at low energies and as $E^{-2.2}$ for energies above 500 GeV.

1. Introduction

For the study of the mechanisms responsible for primary cosmic-ray time variation it is necessary to determine the variation amplitude as well as its dependence on primary energy. This could be achieved by determining the various secondary amplitudes at different energies and correlating them with those of the primary flux.

Up to primary energies sensitive to the geomagnetic field, that is, up to approximately 15 GeV, the correlation between primary and secondary variations could be determined from geomagnetic data (Webber and Quenby 1959, Rao and Sarabhai 1961). At high energies (i.e. $\gg$ 15 GeV), Dorman (1963) applied the nuclear cascade model of Roederer (1954) to calculate the so-called coupling coefficients for the muon component in the vertical direction. Owing to the several approximations used in Dorman's calculations and the complexity involved in the computation of nuclear cascade growth, further modification of this method is needed.

The present work outlines a different treatment for the determination of coefficients of coupling between primary and sea-level muon variations not only in the vertical direction but also at different zenith angles. The method used is based on applying the Cocconi–Koester–Perkins (CKP) model originally introduced by Cocconi *et al.* (1961) to account for nucleon–light-nuclei interactions at accelerator energies (i.e. $<$ 30 GeV). Evidence supporting the applicability of such a model to at least 10^5 GeV has been put forward by various authors (e.g. Brooke *et al.* 1964, Craig *et al.* 1968).

2. Theoretical treatment

The expression given by Dorman (1963) relating the primary and secondary variation amplitudes may be generalized to include zenithal angles other than the vertical. Let $D(E)$ represent the primary differential spectrum and $I(\Delta E_\mu, \theta)$ the muon intensity corrected for meteorological effects and measured at zenith angle θ under absorber thickness ΔE_μ. The general expression for the secondary muon

variation at a location with rigidity R may then be written in the form

$$\frac{\delta I(\Delta E_\mu, \theta)}{I(\Delta E_\mu, \theta)} = \int_R^\infty \frac{\delta D(E)}{D(E)} C(E, \Delta E_\mu, \theta)\, \mathrm{d}E \tag{1}$$

where

$$C(E, \Delta E_\mu, \theta) = \frac{D(E) S(E, \Delta E_\mu, \theta)}{I(\Delta E_\mu, \theta)} \tag{2}$$

is the coefficient of coupling between such variation and the primary one $\delta D(E)/D(E)$. The function $S(E, \Delta E_\mu, \theta)$ represents the integral muon multiplicity or 'yield function' which expresses the number of muons reaching sea level at angle θ with energies greater than ΔE_μ as a result of the interaction of a single primary particle having energy E. If the primary spectrum is represented by the simple power law

$$D(E)\, \mathrm{d}E = cE^{-\gamma}\, \mathrm{d}E \tag{3}$$

it follows that

$$I(\Delta E_\mu, \theta) = c \int_R^\infty S(E, \Delta E_\mu, \theta) E^{-\gamma}\, \mathrm{d}E. \tag{4}$$

For determining the coupling coefficients represented by equation (2), we have adopted the CKP model to derive the integral muon multiplicity at sea level. Following Brooke *et al.* (1964), the CKP relation expressing the number of charged pions of energies $E_\pi + \mathrm{d}E_\pi$ resulting from the interaction of a primary proton having energy E with an air nucleus is written in the form

$$N(E, E_\pi)\, \mathrm{d}E_\pi = \frac{2}{1-(1-K_t)^{\gamma-1}} \frac{A}{T_\pi} \exp\left(-\frac{E_\pi}{T_\pi}\right) \mathrm{d}E_\pi \tag{5}$$

where $A = aE^{1/4}$, $T_\pi = K_\pi E^{3/4}/3a$ and $a = 0{\cdot}45$, K_π and K_t being the fraction of proton energy passed on to the pions and the proton inelasticity, respectively. Accordingly, we obtain the following expression for the sea-level differential muon multiplicity:

$$M(E, E_\mu, \theta)\, \mathrm{d}E_\mu = N(E, E_\pi) g(E_\pi, \theta) W(E_{\mu\mathrm{p}}, \theta)\, \mathrm{d}E_\pi \tag{6}$$

where $g(E_\pi, \theta)$ is the pion decay probability, $W(E_{\mu\mathrm{p}}, \theta)$ the muon probability of survival from production layer to sea level, and $E_{\mu\mathrm{p}}$, the muon production energy, is given by

$$E_{\mu\mathrm{p}} = rE_\pi = E_\mu + d \sec\theta \tag{7}$$

where E_μ is the muon sea level energy, r is a constant and d is the energy lost through ionization processes by a muon travelling to sea level in the vertical direction. The function $g(E_\pi, \theta)$, evaluated for all pion generations, could be determined by comparing the expression for the differential muon spectrum at production

$$I(E_{\mu\mathrm{p}}, \theta)\, \mathrm{d}E_{\mu\mathrm{p}} = g(E_\pi, \theta)\, \mathrm{d}E_\pi \int_R^\infty N(E, E_\pi) D(E)\, \mathrm{d}E \tag{8}$$

with that given by Barrett *et al.* (1952)

$$I(E_{\mu\mathrm{p}}, \theta)\, \mathrm{d}E_{\mu\mathrm{p}} = \frac{B_\pi}{B_\pi + rE_\pi \cos\theta} F(E_\pi)\, \mathrm{d}E_\pi. \tag{9}$$

Since the integral in equation (8) represents the pion production spectrum $F(E_\pi)$, it follows that

$$g(E_\pi, \theta) = \frac{B_\pi}{B_\pi + rE_\pi \cos\theta}. \tag{10}$$

From the above equations we obtain the following expression for the integral multiplicity:

$$S(E, \Delta E_\mu, \theta) = \int_{\Delta E_\mu}^{\infty} M(E, E_\mu, \theta)\, \mathrm{d}E_\mu$$

$$= Kf(E, \theta) \int_{\Delta E_\mu}^{\infty} \exp(-\alpha E^{-3/4} E_\mu) \frac{W(E_\mu, \theta)}{B_\pi + d + E_\mu \cos\theta}\, \mathrm{d}E_\mu \tag{11}$$

where

$$K = 2a\alpha B_\pi \qquad f(E, \theta) = \frac{E^{-1/2} \exp(-\alpha E^{-3/4} d \sec\theta)}{1-(1-K_t)^{\gamma-1}}$$

and

$$\alpha = 3a/rK_\pi.$$

3. Results and discussion

With collision parameters $K_\pi = 0{\cdot}35$ and $K_t = 0{\cdot}47$ the integral multiplicities given by equation (11) have been calculated taking into consideration the dependence of primary spectrum exponent γ on primary energy, as reported by Barrett *et al.* (1952). Computations carried out using an IBM 1130 computer cover the range of primary energies from 1 to 10^6 GeV, and absorber thicknesses ΔE_μ from 0 up to 100 GeV. Figure 1 shows the results of such computations for $\theta = 0°$.

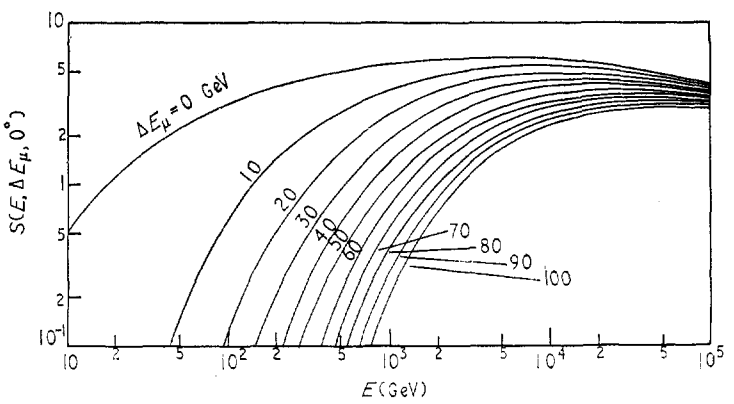

Figure 1. Variation of integral multiplicity with primary energy at different absorber thicknesses.

In order to check the present method, the multiplicities calculated in the region of low energies have been plotted as shown in figure 2, together with those deduced by Webber and Quenby (1959) from geomagnetic data. It can be seen from this figure that the agreement between the two methods is good. The present results have also been compared in the high-energy region with those of Dorman (1963), as shown in figure 3. Although the methods of calculation differ considerably, the results shown

in figure 3 indicate general accordance over a wide range of primary energies. However, it should be noted that, for energies greater than 10^5 GeV, Dorman's multiplicities do not show as much decrease with increasing primary energy as the present results, although such a decrease should be expected on the basis of a pion production function decreasing with increase of average pion energy.

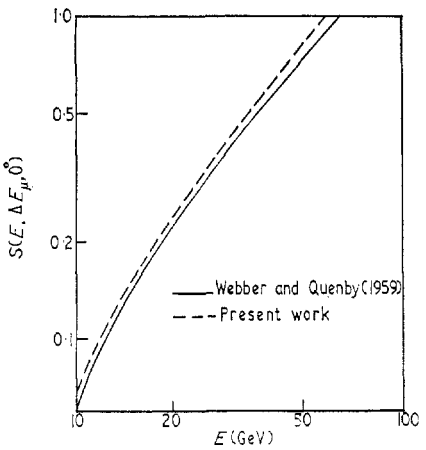

Figure 2. Integral muon multiplicity in the geomagnetic sensitive region. $\Delta E_\mu = 3$ GeV.

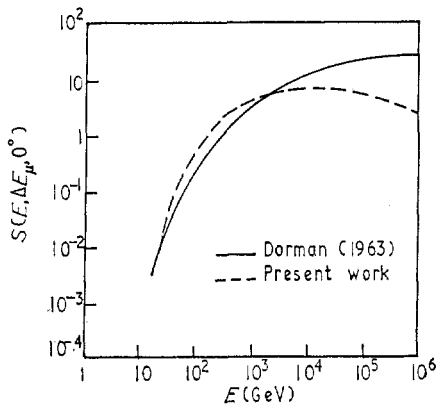

Figure 3. Integral muon multiplicity in the high-energy region. $\Delta E_\mu = 10$ GeV.

From the calculated multiplicities it has been possible to compute the coupling coefficients using equations (2) and (4), and taking for the differential primary spectrum $D(E)$ the data given by Barrett *et al.* (1952). It is found that the coefficients depend weakly on zenith angle. Figure 4 shows the results obtained for $\theta = 0°$.

From the results obtained it could be concluded that, at low primary energies, the coupling coefficients increase as E^3 up to a maximum $C_{\max}$ at a primary energy given approximately by

$$E(C_{\max}) = 6\Delta E_\mu \tag{12}$$

where $C_{\max}$ may be related to the absorber thickness ΔE_μ, in the ran e

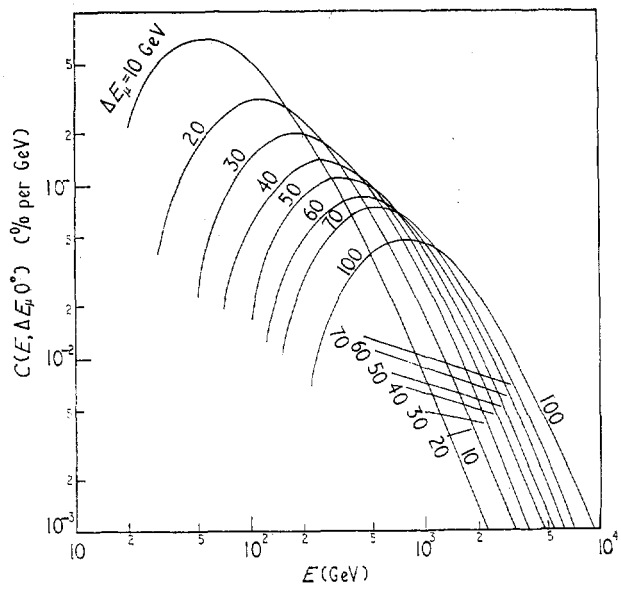

Figure 4. Coupling coefficients for muon component for various thicknesses of absorber.

$10 < \Delta E_\mu < 100$ GeV, by the power law

$$C_{\max} = 10(\Delta E_\mu)^{-1\cdot 17}. \tag{13}$$

Beyond $E(C_{\max})$ the coefficients decrease with increasing primary energy according to the power law

$$C(E, \Delta E_\mu) = A(\Delta E_\mu)E^{-2\cdot 2} \qquad (E > 500 \text{ GeV}) \tag{14}$$

where $A(\Delta E_\mu)$ is a parameter nearly independent of E and increases with increasing absorber thickness ΔE_μ.

References

Barrett, P. H., *et al.*, 1952, *Rev. mod. Phys.*, **24,** 133–78.

Brooke, G., Hayman, P. J., Kamiya, Y., and Wolfendale, A. W., 1964, *Proc. Phys. Soc.*, **83,** 853–69.

Cocconi, G., Koester, A., and Perkins, D. H., 1961, *Int. Conf. on Theoretical Aspects of Very High Energy Phenomena* (Geneva, CERN), p. 128.

Craig, R., Osborne, J. L., Wolfendale, A. W., and Young, E. C. M., 1968, *J. Phys.* A: *Gen. Phys.*, **1,** 61–71.

Dorman, L. I., 1963, *Variations of Cosmic Rays and Study of Cosmos*, Moscow, English Transl. by USAF, FTD-MT-64-86, 1966, p. 210.

Rao, V. R., and Sarabhai, V., 1961, *Proc. R. Soc.* A, **263,** 101–17.

Roederer, J. G., 1954, *Z. Naturforsch.*, **9a,** S., 740.

Webber, W R., and Quenby, J. J., 1959, *Phil. Mag.*, **4,** 654–64.