It is helpful to impose one more restriction on the discussion: since it is not yet known how to calculate in higher orders of perturbation theory for the weak interaction, this material is valid only if the next term for (7) (which should be proportional to G^4E^6) is negligible. When this is so,

$$1 \geqslant G^2F^2(-E^2)E^4k^2/\pi \tag{8}$$

on dimensional grounds, and $k = 1$ if (5) holds.

The magnitudes of cross-sections for the weak processes (1) relate to electromagnetic measurements of F. From fits to data recorded at small values of q^2,

$$F(-q^2 = E^2) \sim A + Bn(m^2 + E^2)^{-1} \tag{9}$$

appears to be the asymptotic behaviour of F, where A represents a hard-core part and the second term is contributed by n mesons of mean mass near m. If (9) is maintained with $A = 0$, then (7) satisfies both the inequalities (2) and (8), but is asymptotically too small to support (1) as viable high-energy reactions. If A is non-zero, the inequalities cease to hold for E near $10^{12}A^{-\frac{1}{2}}$ eV. In view of the present effective limit[5] on A, this is also not very satisfactory.

However, if it is assumed that F obeys (6) initially but is constant above some critical value $-E_c^2$ of its argument, and that $K(x - y)$ is not given by (5), a significant value of σ can be obtained. The second assumption is necessary because, if (3) is a precursor of (1), then (8) is violated for primary proton energies above about 10^{17} eV, while it has been argued that the highest-energy primary on record,[6] at 10^{20} eV, is possibly a proton.[7, 10]

Let us now assume that

$$K(x - y) = \sum_{n=0}^{\infty} (E_c^{2n}d_n)^{-1} \Box^n \delta(x - y), \tag{10}$$

where

$$\Box = \frac{\partial^2}{\partial t^2} - \nabla^2 .$$

The interpretation of (10) is that the low-energy weak interaction, which seems like a point interaction, shows a non-locality with a scale of length of E_c^{-1} in natural units at high energies. Then the result $\Box^n j_\alpha = q^{2n} j_\alpha$ can be obtained from the definition of j_α, and the cross-section (7) becomes

$$\sigma = (1 + r^2)G^2F^2(-E^2)k^2(E/E_c)E^2/2\pi , \tag{11}$$

where

$$k(E/E_c) = \sum_{n=0}^{\infty} (-E^2/E_c^2)^n \, d_n^{-1}$$

and, according to (6) and the first assumption,

$$F^2(-E^2) = 4(\sqrt{2} + E_c^2)^{-4}$$

for $E < E_c$. The choice of d_n is free to the extent that the distribution (10) has the desirable Lorentz-invariant properties established by Efimov;[8] $d_n = (2n)!$ gives $k(x) = \cos x$, $d_n = (2n + 1)!$ gives $k(x) = x^{-1} \sin x$, and $d_n = 2^{2n}(n!)^2$ gives $k(x) = J_0(x)$. Nevertheless, all the choices lead to the same qualitative physical behaviour for large x. With the choice of the Bessel function, and under the restrictions of (2) and (8), the asymptotic form of (11) up to 10^{20} eV is

$$\sigma(E) \sim \frac{4(1 + r^2)}{\pi^2} \, \frac{G^2EE_c}{(\sqrt{2} + E_c^2)^4} \cos^2\left(\frac{E}{E_c} - \frac{\pi}{4}\right)$$

for $E_c > 5.6 \times 10^9$ eV.

It remains to conclude a detailed model of strong primary cosmic-ray processes which can generate large numbers of antinucleons in the reaction (3), and to examine a conflict between the folklore that strong cross-sections are constant from accelerator energies to cosmic-ray energies and a suggestion[9] that this need not be so.

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A New Determination of Muon Directional Coupling Coefficients

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Primary cosmic rays passing through the solar system carry with them valuable information about solar and astrophysical phenomena in the form of intensity and spectral variations. In order that this information be efficiently extracted from observations of the directional cosmic-ray flux at the surface of the Earth, it is essential to have accurate information available to enable the relating of the observed secondary cosmic-ray directions of motion and intensity to those outside the range of the disturbing terrestrial influences.

The so-called coupling coefficients constitute one of these requirements, expressing the contribution to the counting rate of a detector for primaries in any portion of the primary particle spectrum. They may conveniently be expressed in the form

$$W_i(\theta, P) = -100(\mathrm{d}N/\mathrm{d}P)/N \ \%(\mathrm{GeV}/c)^{-1}$$

for the component i at a zenith angle θ. W is the coupling coefficient, N the total counting rate of a detector and P the momentum in GeV/c.

It has become evident that incomplete knowledge of the muon coupling coefficients is holding back in some major respects the progress in detailed interpretation of cosmic-ray telescope data. Theoretical estimates of the coefficients prove inordinately difficult at all but high momenta, and the experimental determinations to date for low values of momentum (involving correlation of observed muon intensity with the calculable primary cut-off momenta in the terrestrial magnetic field) fail to agree satisfactorily (e.g. the results of Dorman,[1] and Webber and Quenby,[2] for the vertical flux).

In an attempt to resolve this situation, a new determination of the low-momentum coefficients has been made, employing techniques in the analysis designed to avoid the difficulties encountered in earlier estimates. The preliminary results of this work are reported briefly here.

A latitude survey of directional meson intensity was carried out during the period January to August 1967, when observations were made at seven sites along the eastern Australian coast, from Hobart (Tasmania) to Mossman (North Queensland). Two telescope arrays were used, containing a total of eleven narrow-angle geiger-counter telescopes, directed to zenith angles of 0°, 22°.6, 45°.2 and 67°.8, and each containing 10 cm of lead absorber. The data were corrected for air-shower contamination, and also to remove the high-latitude east-west asymmetry effect (produced as a result of systematic changes in the path length of secondary particles through the atmosphere due to magnetic deflection). Results given by Fenton[3] were referred to for the magnitude of the asymmetry for various magnetic-field strengths and zenith angles, and a cosine dependence was assumed for the asymmetry on azimuth with respect to magnetic west, as evident in earlier results reported by one of us (Cooke[4]).

The corrected results were converted into the form inclined/vertical ratio—the intensity in a particular direction compared with that observed by the same telescope vertically directed at the same site.

Since the muon atmospheric pressure and temperature coefficients are very nearly constant for all zenith angles, the inclined/vertical ratio may be taken to be dependent only on the value of the geomagnetic cut-offs in the directions concerned and on the essential zenith dependence of meson intensity. At a given zenith angle at any site, the variation of the ratio with azimuth angle reflects the influence of the cut-offs on the inclined telescope counting rate.

Theoretical calculations of proton cut-off momenta (the minimum required for a particle to penetrate through the magnetic field and arrive in a particular direction at a particular site) were carried out for the observing hemisphere over each of the survey sites except Hobart.

The calculated mean cut-off over the telescope acceptance cones for each observing direction (calculated using the computer programme described later in this paper) was plotted against the observed ratio value such as in Figure 1. It is apparent that the results for the different sites are displaced vertically. If the vertical cut-offs were the same at each site, then the inclined/vertical ratio would be expected to be the same for telescopes pointing in directions where the inclined cut-offs are the same.

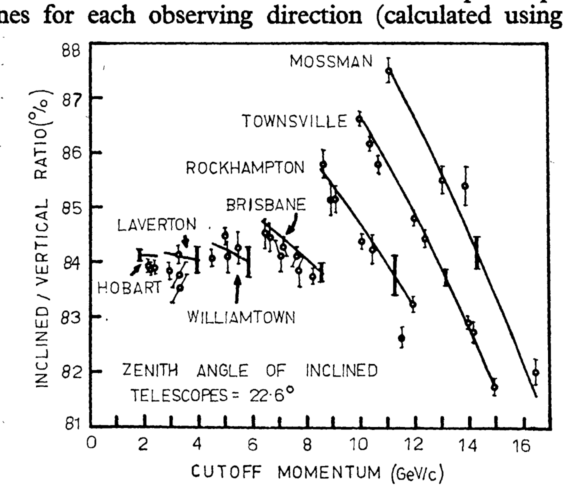

Figure 1. Experimental values of inclined/vertical ratio plotted against mean cut-off momentum in the inclined direction. Single error bars are the standard deviation errors on the individual observational points. The double error bars are the telescope calibration standard deviation error, common to all measurements at any site.

The vertical displacement, then, is produced by the change in vertical cut-off value. From the magnitude of the vertical displacement, the change in vertical intensity from site to site can be directly calculated and is independent of atmospheric changes with latitude (see Figure 2).

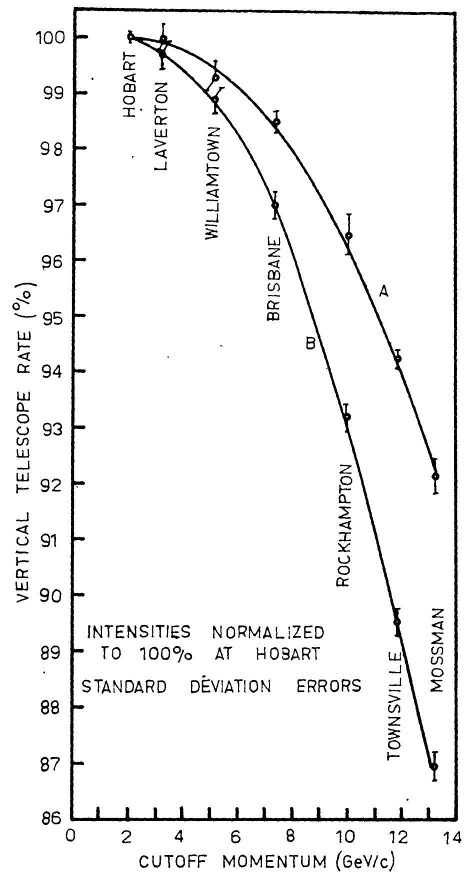

Figure 2. Variation of vertical narrow-angle telescope rate with latitude variation of mean cut-off momentum. The set of points A was obtained using the ratio method (the smooth line is the fitted intensity-cut-off relationship), whilst set B is the actual pressure-corrected variation of intensity observed. The difference between the sets is due to the upper atmospheric temperature effect. Error bars are standard deviation errors.

A single expression has been found to represent all the observational data. It relates the relative counting rate, I, of a detector to the cut-off momentum, P GeV/c, at any zenith angle θ and azimuth angle ϕ to the counting rate at that zenith angle if the atmospheric cut-off P' exceeds the geomagnetic cut-off P. For $P \leqslant P'$ the secondary particle intensity is azimuth independent, apart from the high-latitude east-west asymmetry effect mentioned earlier.

$$\begin{aligned} I(\theta,\phi,P) &= 100 - k_\theta(P(\theta,\phi) - P'(\theta))^2\ \% && \text{for } P > P' \\ &= 100\% && \text{for } P \leqslant P', \end{aligned}$$

where P', the atmospheric cut-off momentum, is the minimum muon momentum required to penetrate the atmosphere at the zenith angle θ, k_θ is a zenith dependent constant—closely represented by exp

$$(-0.850 \exp \theta^{1\cdot 7} - 1.838).$$

In fitting this relationship, a computer programme was used which calculated the theoretical inclined/vertical ratios by integrating contributions to the counting rate of the telescope from all parts of the acceptance cone, taking into account the geomagnetic cut-off values (including penumbral bands), meson zenith dependence, and the atmospheric cut-offs. By comparing such calculations with the results computed for hypothetical constant cut-off sites, accurate mean values of cut-off for each observing direction were estimated. In Figure 1, the theoretical values are plotted for comparison with the observed values.

The coupling coefficients $W(P)$ are

$$W(P) = -100 \left(\frac{dN}{dP}\right) \Big/ N$$

$$= -\frac{dI}{dP}\,\%(\text{GeV}/c)^{-1},$$

so that
$$W(P) = 2k_\theta(P - P') \text{ for } P > P'$$
$$= 0 \qquad \text{for } P \leqslant P'.$$

This expression applies only to the range of P for which muon intensity observations were made.

The experimental coefficient values have been plotted in Figure 3 for four zenith angles. Since spectral changes are small for the primary flux with $P > 5$ GeV/c, these coefficients may be used without appreciable error for any time in the solar cycle.

The gross-yield functions, or average number of detected secondaries produced by a primary of particular momentum (ignoring contributions from particles heavier than protons), have been plotted in Figure 4. These are values of the ratio

$$\frac{d\mathcal{J}}{dP} \Big/ \frac{dN}{dP}$$

where $d\mathcal{J}/dP$ is the differential primary proton spectrum. The primary spectrum of Ormes and Webber[5] was used for this purpose.

Yield function curves like these, taking into account muon production by heavier nuclei, will be used to extrapolate the experimental results to meet the high momentum theoretical values (subject to the condition $\int_0^\infty W(P)dP = 100\%$) to obtain a complete set of coupling coefficients.

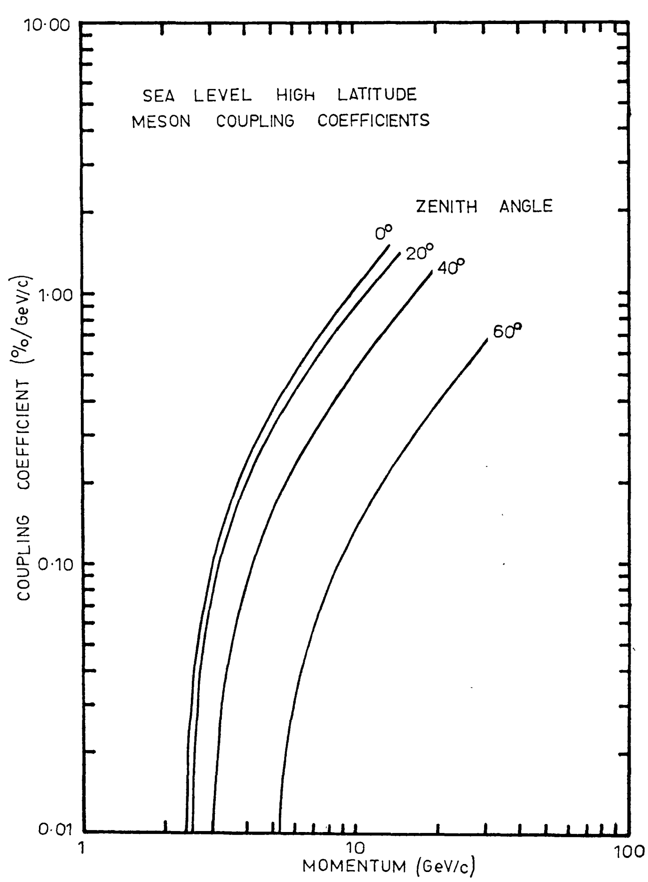

Figure 3. Experimental high-latitude sea-level muon coupling coefficients.

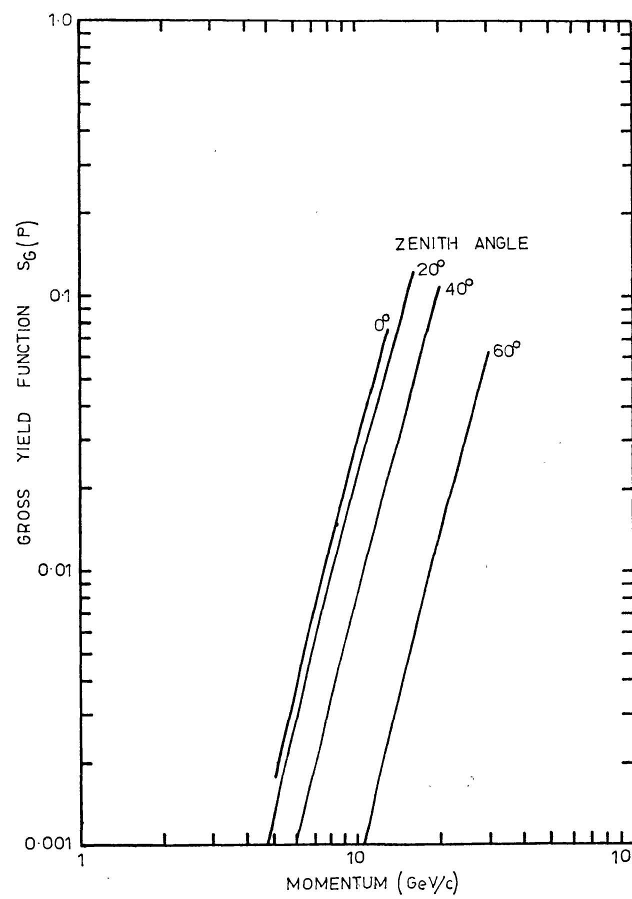

Figure 4. Experimental sea-level muon gross-yield functions.