which would be measured by a $1/v$ detector would occur not far from the maximum of the nucleonic cascade and would be expected to vary in the same way, with geomagnetic latitude. With the above reasoning in mind, a parabola was fitted to the six points by the method of least squares (see Fig. 1). It can be seen that the curve falls within all of the estimated probable errors.

An interesting point about the above theory is that the geomagnetic cutoff (E_{min}) is different for primary protons than for alpha particles. This might seem to predict the existence of a double maximum in the neutron intensity curves due to nucleonic cascades which are originated by these different primaries. A close look at the experimental data, particularly those which were obtained at $\lambda=10.1°$ where the effect would be more pronounced, indicates that perhaps such an effect does exist and what we commonly think of as the neutron intensity maximum may be the composite of two such maxima. More accurate data in the vicinity of 100 mb are needed before anything further can be said about this point.

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Cosmic-Ray Electrons Near Sea Level and at Mountain Altitudes*

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A cloud chamber containing aluminum and lead plates has been used to study the intensities and energies of cosmic-ray electrons as a function of altitude and zenith angle. The energy distribution is found to be approximately independent of altitude and zenith angle. The decrease of electron intensity with zenith angle is found to be much less than has been reported by investigators using counter telescopes. The increase of the vertical intensity with altitude is found to be slightly smaller than previously reported. The numbers of electrons resulting from decay and collision processes of μ mesons have been calculated and subtracted from the observed numbers. The residual electrons have an exponential altitude dependence with an absorption length of 135 ± 15 g cm^{-2}, and a zenith-angle dependence which is much less steep than would be expected if they preserved the directions of the primary particles from which they originate.

INTRODUCTION

OBSERVATIONS of cosmic-ray electrons are of interest in order to provide information for comparison with suppositions concerning the origin of electrons in the atmosphere. At sea level, most of the observed electrons can be explained in terms of decay and collision processes of μ mesons. At higher altitudes electrons resulting from nuclear interactions, primarily through the decay of π^0 mesons, become increasingly important.

A number of measurements have been made of the intensities and energy distributions of electrons at various altitudes and zenith angles,[1–5] with results which are not entirely consistent, especially with regard to the variations with zenith angle. The most extensive surveys[1,3] were made with counter telescopes and absorbers. In this way, good statistical accuracy can be obtained but accurate identification of individual events is not possible. In order to obtain better information about the electron intensities, the present experiment was undertaken. A cloud chamber containing metal plates and triggered by a narrow-angle telescope was used. This method has several advantages over methods using counters alone. All electrons which reach the visible region of the chamber can be counted, regardless of how they scatter in the plates. Side showers can easily be recognized. Energetic electrons can be distinguished from heavier particles by the showers they produce in a series of lead plates, and their energies can be estimated from the development of the showers. A few aluminum plates, above the lead plates, serve to stop slow electrons, whose energies can be determined from their ranges. Stopped heavy particles can be recognized by their large ionization. With this type of apparatus, measurements were made at Ann Arbor (altitude 280 m), Echo Lake (3260 m), and Mt. Evans (4300 m).

APPARATUS

The main body of the cloud chamber was a Pyrex cylinder of inside diameter 29 cm. The average thickness in the region below the counters was 0.63 cm. Eight absorbing plates were used in the chamber. The top plate was 0.32 cm of aluminum (alloy 24S), the

* This work is reported in greater detail in a dissertation submitted in partial fulfillment of the requirements for the Ph.D. degree at the University of Michigan (1952). The work was supported in part by the joint program of the Office of Naval Research and the U. S. Atomic Energy Commission, and most of it was done while the author held an AEC Predoctoral Fellowship.

[1] K. I. Greisen, Phys. Rev. **61**, 212 (1942).
[2] K. I. Greisen, Phys. Rev. **63**, 323 (1943).
[3] E. D. Palmatier, Phys. Rev. **88**, 761 (1952).
[4] Lovati, Mura, Succi, and Tagliaferri, Nuovo cimento **12**, 526 (1954).
[5] C. N. Chou and M. Schein, Phys. Rev. **98**, 162 (1955).

next two were each 0.64 cm of aluminum, the fourth was 0.66 cm of brass, and the bottom four were each 0.64 cm of lead. A thin bronze foil was fastened to the top plate and four short vertical wires were soldered to the foil. These wires formed a rectangle outlining the region which could be traversed by particles passing through the counters and upper wall of the chamber without scattering. The rectangle measured 4.0 by 10.5 cm, with the short dimension along the line of sight of the camera. The depth of focus of the camera was about 10 cm. The region of good illumination was considerably larger than the region which could be traversed by unscattered particles.

The chamber was triggered by an array of nine Victoreen 1B85 counters. These counters are made of aluminum, with wall thickness 0.030 g cm^{-2}, active length 6.4 cm, and diameter 1.9 cm. In order to keep the beam defined by the counters as close as possible to the focal plane of the camera, the counters were arranged as indicated in Fig. 1, with their axes parallel to the front of the chamber. The three counters on the right were connected to one triple-coincidence circuit, the three in the middle to another, and the three on the left to a third. The resolving time was about 35 μsec. The counters were mounted on a light-weight wooden framework, all of which was well outside the beam. The experiments were performed in wooden buildings beneath slots covered with thin aluminum (0.09 cm at Ann Arbor, 0.05 cm in Colorado).

The camera, cloud chamber, and counters were all fixed rigidly to a frame which could be rotated about an axis parallel to the axis of the chamber and could be clamped solidly at any angle up to 60° from the vertical. Measurements were made at angles of 0°, 45°, and 60°. At Ann Arbor, the apparatus was tilted toward the east, at Echo Lake toward the north, and at Mt. Evans toward the southeast. These directions were determined by the orientations of the buildings used.

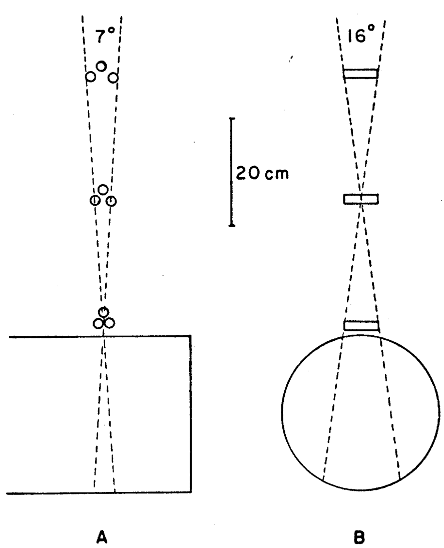

FIG. 1. Arrangement of counters and cloud chamber. A. Side view. B. Front view.

EXPERIMENTAL RESULTS

The negative of each photograph was examined carefully through a stereoscopic viewer. Those pictures in which conditions were not suitable for the formation of tracks in all sections of the chamber were rejected. The only exception is a series taken at 0° on Mt. Evans, in which the uppermost section showed an excessive amount of fog but conditions were good in the rest of the chamber. In these pictures, particles penetrating the top plate were counted but no attempt was made to count those stopping in the top plate.

Each of the acceptable photographs was assigned to one of the following classifications:

Blank pictures show no tracks entering the chamber. Besides events caused by random coincidences and diffuse side-showers, this group includes particles of very low energy which cannot penetrate the wall of the chamber, and electrons which are scattered through large angles in the wall.

Side showers include all events in which two or more particles enter the chamber from a direction other than that defined by the counter telescope.

Extraneous particles include all single particles which do not emerge from the chamber wall within the allowed area beneath the counters and all particles except slow electrons which enter through the allowed area but do not have the right direction. The allowed area is defined by assuming that there is no scattering in the counters.

Penetrating particles pass through the absorbing plates without producing showers, and therefore must be mesons or heavier particles. A penetrating particle, in traversing the lead, frequently produces one or more collision electrons, but such events are easily distinguished from electron-induced showers by the fact that the heavy particle continues on its way without appreciable scattering. Events where the identification is uncertain are extremely rare. To penetrate all the plates μ mesons must have a minimum momentum of 150 Mev/c and protons a minimum of 600 Mev/c.

Stopped heavy particles can be distinguished from electrons by ionization greater than twice minimum with no scattering in the gas.

Slow electrons are those stopping in one of the first four plates. Each was listed according to the plate in which it stopped, and was counted even when it emerged from the top wall of the chamber in a direction different from that defined by the counters. The rms angles of scattering in the chamber wall were estimated from the formula given by Rossi and Greisen[6] for thin layers. Electrons which entered at angles much larger than the expected rms values were recorded as

[6] B. Rossi and K. Greisen, Revs. Modern Phys. 13, 240 (1941).

TABLE I. Observed numbers of electrons and other events. N_s is the total number of track segments beneath the bottom five plates. Numbers followed by the symbol (?) indicate events whose identification is uncertain.

	Ann Arbor			Echo Lake			Mt. Evans			
	0°	45°	60°	0°	45°	60°	$0_{(1)}$°	$0_{(2)}$°	45°	60°
Electrons										
Entering chamber	323	324	285	504	428	341	...	453	491	398
	19?	18?	20?	16?	14?	19?		12?	20?	19?
Penetrating 1st plate	301	294	267	486	402	301	348	435	466	373
	15?	12?	15?	14?	10?	14?	16?	10?	15?	10?
Penetrating 2nd plate	269	253	234	443	342	261	327	408	420	320
	13?	12?	11?	13?	9?	12?	6?	9?	15?	10?
Penetrating 3rd plate	248	220	202	396	304	227	284	373	364	273
	12?	10?	10?	13?	9?	10?	6?	9?	14?	6?
Penetrating 4th plate	158	132	128	271	198	147	182	263	243	176
	8?	8?	9?	11?	6?	7?	6?	8?	9?	2?
$N_s \geq 4$	69	54	51	100	82	57	63	113	98	66
	4?	2?	5?	6?	2?	2?	3?	5?	4?	1?
$N_s \geq 6$	39	37	37	73	55	32	35	81	64	50
	2?	1?	5?	3?			1?	4?	3?	
$N_s \geq 10$	23	20	19	37	30	23	15	49	35	21
			4?	1?			1?	3?	1?	
$N_s \geq 16$	12	12	8	21	13	10	4	22	15	8
			2?					2?		
Multiple el.	36	3	0	50	8	0	31	95	19	3
Other events										
Penetrating particles	1022	1116	905	622	759	625	349	464	607	579
	2?	3?	6?	1?	1?	2?	5?	8?	2?	3?
Total acceptable pictures	1466	1625	1393	1312	1463	1453	878	1114	1436	1443
Blank pictures	58	83	114	70	119	152	93	67	137	169
Side showers	13	30	30	34	84	222	18	27	90	177
Extraneous particles	5	37	24	25	19	31	8	20	36	43
Stopped heavy particles	13	9	9	19	32	56	32	34	41	47
Total counting rate (min^{-1})	0.158	0.066	0.0374	0.400	0.179	0.095	0.560		0.288	0.127

questionable. The error caused by missing electrons which scatter through large angles was estimated to be negligible for those which penetrate the second plate.

Fast electrons include all those penetrating the first four plates. For each of these the total number of track segments beneath the brass plate and beneath each of the lead plates was recorded. In cases where a track was obviously scattered back from the upper surface of a lead plate, it was counted only once. In doubtful cases such tracks were frequently counted twice.

Multiple electrons occurred when air showers were incident in the direction defined by the counters. In such cases several electrons generally enter the chamber within the allowed region, and it is impossible to determine how many of them passed through the counters. The best that can be done in counting these electrons is to set an upper limit by including all those which might be allowable.

A few nuclear interactions were observed in the plates, but these are of no particular interest since it was often impossible to tell whether the counters were triggered by the initiating particle or by an upward-moving secondary.

The observed numbers of electrons and other events are listed in Table I. For Mt. Evans the column headed $0_{(1)}$° gives the data obtained while the top section of the chamber was foggy and $0_{(2)}$° gives the data obtained after this condition was corrected. The "blank pictures" listed under $0_{(1)}$° include electrons stopping in the first plate. The numbers followed by question marks refer to events whose identification is uncertain. Usually the uncertainty has to do with whether the particle went through all the counters.

The total counting rates were measured during part of each run and are included in Table I. The standard deviations are about 3%.

ENERGY CALIBRATION

Slow Electrons

In determining the relationship between range and energy for the slow electrons, it was desired to take into account the fact that, when an electron transfers most of its energy to a photon, the photon may materialize in one of the lead plates and produce one or more observable electrons. In such cases, the electron was counted as having emerged from the plate in which it stopped, and as having traveled a further distance which was estimated by observing the secondary electrons. A rough estimate indicates that a 20-Mev photon passing vertically through the lead plates would have a probability slightly over 0.5 of producing an observable secondary. Therefore, only that part of the radiation loss was counted which involves energy transfers of less than 20 Mev. This partial radiation loss was calculated from the cross sections given by Heitler[7] and by Wheeler and Lamb,[8] with corrections for incomplete screening as indicated by Heitler. The inverse of the sum of this radiation loss and the corresponding collision loss was integrated graphically to give the partial range down to 5 Mev. From the data of Fowler *et al.*[9] and of Hereford and Swann,[10] the average range of a 5-Mev electron, including the effects of multiple scattering, was taken to be 0.72 cm in aluminum and 0.20 cm in brass. The total ranges, obtained by adding the partial ranges to the ranges at 5 Mev, are not corrected for the effect of scattering at energies above 5 Mev because no experimental evidence could be found concerning the penetration probabilities when the electrons are not considered as lost after scattering through small angles. The ranges obtained for aluminum agree to within a few percent with the average ranges given by Fowler *et al.*[9] on the basis of a semiempirical calculation of the effects of scattering.

Table II gives the total amount of material traversed by electrons penetrating the various absorbing plates and the corresponding energies as derived from the above considerations. The Pyrex of the chamber wall was converted to an equivalent thickness of aluminum by comparison of the total energy loss at 15 Mev.

TABLE II. Energy required by electrons to penetrate absorbers.

Penetration	Material traversed	Energy (Mev)
Chamber wall	0.73 cm Al	5.1
1st plate	1.08 cm Al	6.9
2nd plate	1.70 cm Al	10.4
3rd plate	2.32 cm Al	14.4
4th plate	2.32 cm Al+0.66 cm brass	30

Fast Electrons

A common way of estimating the initiating energy of a shower produced in a series of lead plates has been by counting the number of electrons at the maximum of the shower, making no use of information which might be derived from observation at points other than the maximum. This method is not very satisfactory for energies where the maximum number of electrons is small and therefore subject to large fluctuations. The ideal quantity to observe would be the area under the shower curve which gives the number of electrons as a function of depth. Neglecting the effect of scattering, this area is proportional to the initiating energy. The area cannot be determined exactly by experimental methods, but the integral from zero to a depth corresponding to the total amount of absorber present can be approximated by adding together the numbers of electrons observed in each of the spaces between the plates.

The expected average development of showers in lead was obtained from Wilson's Monte Carlo results.[11] Wilson gives two measures of the number of electrons at a given depth: n_{st}, the number observed near the shower axis and within 30° of the original direction, and n_c, the number observed when all electron paths between successive lead plates are counted. The electrons counted in the present experiment were essentially those scattered by less than 90°, and a value halfway between n_{st} and n_c was used. The material above the lead was considered equivalent to 0.5 radiation lengths of lead, and the numbers of electrons expected beneath the brass plate and beneath each of the lead plates were obtained from the Monte Carlo curves and added together. The results were plotted as a function of energy from 50 Mev to 500 Mev, and the energies corresponding to various integral numbers of track segments (Table III) were obtained from the graph.

A recent study of showers produced by photons from π^0 decay[12] indicates that the Monte Carlo calculations are more accurate when used with a higher cutoff, giving smaller numbers of electrons. Therefore it is probable that the energies listed in Table III are too small. Other unpublished observations[13] also indicate that these energies may be too small.

ERRORS AND CORRECTIONS

Scattering

For electrons of energy greater than about 10 Mev, the effect of scattering in the chamber wall is negligible. Scattering in the roof, the top counters, and the bottom counters has a negligible effect at even smaller energies. The most serious source of error is scattering in the central counters. Electrons which should be counted will be eliminated if they scatter through small angles

[7] W. Heitler, *The Quantum Theory of Radiation* (Oxford University Press, New York, 1947), p. 172.
[8] J. A. Wheeler and W. E. Lamb, Phys. Rev. **55**, 858 (1939).
[9] Fowler, Lauritson, and Lauritson, Revs. Modern Phys. **20**, 236 (1948).
[10] F. L. Hereford and C. P. Swann, Phys. Rev. **78**, 727 (1950).
[11] R. R. Wilson, Phys. Rev. **86**, 261 (1952).
[12] P. A. Bender, Nuovo cimento (to be published).
[13] A. M. Shapiro, private communication, 1951.

TABLE III. Energies of showers in the lead plates. A shower of n track segments corresponds to an energy of E Mev.

n	4	6	10	16
E	80	125	230	395

in such a way that they miss the bottom counter. There is only partial compensation by others scattered in, because these are required to pass through the top counter. Comparison of the rms angle of scattering and the angle intercepted by the diameter of one of the bottom counters viewed from the position of the central counters indicates that the two are equal at about 10 Mev. Therefore it may be expected that the effect becomes negligible only at energies considerably above 10 Mev. Unfortunately, the importance of this effect was not realized at the time the apparatus was designed.

The effect of scattering in the central counters was calculated in an approximate way by considering the counters as flat plates of thickness πd, where d is the thickness of the counter walls and πd is the average path length through the two walls for particles distributed uniformly in space and moving perpendicular to the axis of the counter. This approximation gives an over-correction because most of the trajectories of unscattered particles pass through the middle of the central counters where the thickness is less than πd, while most of the trajectories of particles that can be scattered into the bottom counter pass near the sides where the thickness is greater.

The results of the calculation can be expressed in terms of the tabulated[14] functions $h_0(x)=\int_x^\infty \exp(-y^2/2)dy$, $h_1(x)=\int_x^\infty h_0(y)dy$, and $h_2(x)=\int_x^\infty h_1(y)dy$. A Gaussian angular-distribution function, as given by Rossi and Greisen[6] was used. It was found that in a plane perpendicular to the axes of the counters the fraction of electrons of energy E which are scattered out is

$$F_1=2(\lambda E)^{-1}(2\pi)^{-\frac{1}{2}}[h_1(0)-h_1(\lambda E)],$$

where $\lambda=4a(2\pi d)^{-\frac{1}{2}}(21L)^{-1}$, a is the diameter of a counter, L is the distance between the top and bottom counters, and d is the thickness of a counter wall in radiation lengths. It is assumed that $a\ll L$. In the present case, $\lambda=0.0936$ Mev^{-1}. The number of electrons which, after entering along trajectories that do not pass through the bottom counter, are deflected into the bottom counter is given, in terms of the number entering along allowed trajectories, by

$$F_2=F_1-2(\lambda E)^{-2}(2\pi)^{-\frac{1}{2}}[h_2(0)-2h_2(\lambda E)+h_2(2\lambda E)].$$

Thus, the number of electrons counted is reduced by the fraction (F_1-F_2). There is a small loss due to scattering in the perpendicular plane, past the ends of the counters, but above 10 Mev the effect is negligible in the present case. The observed differential energy distribution should be multiplied by the correction factor $C(E)=(1-F_1+F_2)^{-1}$. If $S_0(E)$ is the observed integral distribution, the corrected integral distribution is given by $S(E)=S_0(E)+\int_E^\infty S_0(U)C'(U)dU$. In general, the integral must be evaluated numerically.

Other Experimental Errors

Since the counting rates were not determined with great accuracy, the data were analyzed in terms of the ratios of the numbers of electrons and penetrating particles observed. Accurate values of the intensities of penetrating particles are known from other experiments.[1,15] With this method of analysis the small errors due to counter inefficiency and meteorological variations are completely eliminated.

A particle which passes through only two of the counters may, by collision in the roof or one of the counters, produce a secondary electron capable of triggering the third counter. By comparison with the measurements of Brown *et al.*[16] it is estimated that the number of such events in which an electron enters the chamber in such a way as to be counted is completely negligible.

It is estimated that one or two percent of the measured electron intensity may consist of photons which materialize in the roof or in the top counters. This error is small compared to the statistical errors and no correction has been made for it.

Another possible source of error is the appearance in the cloud chamber of electrons which did not pass through the counters. To find how many of these might originate from random coincidences, 500 pictures of penetrating particles (where it is quite certain what triggered the counters) were examined, and only one electron was found. Comparison with the observed numbers of blank pictures and extraneous particles indicates that errors due to random coincidences are negligible. The number of false events where the observed electrons are related to the particles that triggered the counters is difficult to determine but should be only a small fraction of the number of blank pictures and extraneous particles. Because of the possibility of this type of event, it is necessary to use a triple- rather than a double-coincidence telescope, although the latter appears more desirable in order to reduce the effects of scattering. A preliminary run at Ann Arbor, without the central counters, gave eight times the rate of blank pictures reported here and indicated an energy distribution with relatively more electrons of low energy. This surplus of slow electrons is interpreted as the result of including events where the observed electron did not pass through the whole telescope. Such errors are most likely among the electrons of very low energy, which were counted even when they entered the chamber at large angles.

[14] *Mathematical Tables* (British Association for the Advancement of Science, London, 1931), Vol. 1.

[15] W. L. Kraushaar, Phys. Rev. **76**, 1045 (1949).

[16] Brown, McKay, and Palmatier, Phys. Rev. **76**, 506 (1949).

RELATION BETWEEN COUNTING RATE AND INTENSITY

The relation between the observed counting rates and the directional intensities depends on the variation of zenith angle over the telescope. The arrangement here is different from usual in that the axes of the counters remain in a vertical plane when the telescope is tilted. The relation was calculated under the assumption that the variation of intensity with zenith angle is given by $I(\theta)=I_0\cos^n\theta$. With two counters of diameter a and length b, separated by a distance L ($L\gg a$), one can show that the counting rate when the axis of the telescope is at an angle θ to the vertical is $N(\theta)=I(\theta)a^2\alpha^2(1+C\alpha^2)$, where $\tan\alpha=b/L$ and $C=1/3-n/12+[n(n-1)/12]\tan^2\theta$. Terms involving higher powers of α have been neglected. With the three sets of counters used in the present experiment, the ratio of counting rate to absolute intensity is $3a^2\alpha^2(1+C\alpha^2)$, where $3a^2\alpha^2=0.21$ cm² sterad. For $n=3$, the correction factor, $1+C\alpha^2$, is 1.002 at 0°, and 1.031 at 60°. The effect is less for smaller values of n. No components with $n>3$ were observed. Since the statistical uncertainties are rather large, the above correction factor has been neglected.

INTENSITIES OF PENETRATING PARTICLES

The counting rates of penetrating particles, obtained by multiplying the numbers of penetrating particles by the total counting rates and dividing by the numbers of acceptable pictures, are given in Fig. 2. The lines are drawn with a slope corresponding to $\cos^{2.1}\theta$, in agreement with the results obtained by other investigators.[1,15] The agreement is satisfactory since the counting rates were not measured with extreme care. The determination of counting rate for 45° at Ann Arbor exhibits poor internal consistency and may include a systematic error.

The vertical intensity of penetrating particles at Ann Arbor is found to be 0.0087 ± 0.0003 cm^{-2} sterad^{-1} sec^{-1}. The best measurement of this quantity is probably that of Kraushaar[15] who found the vertical intensity at Ithaca to be 0.00904 ± 0.00005 cm^{-2} sterad^{-1} sec^{-1} (corrected to a minimum range of 23 g cm^{-2} of air by means of Kraushaar's range spectrum).

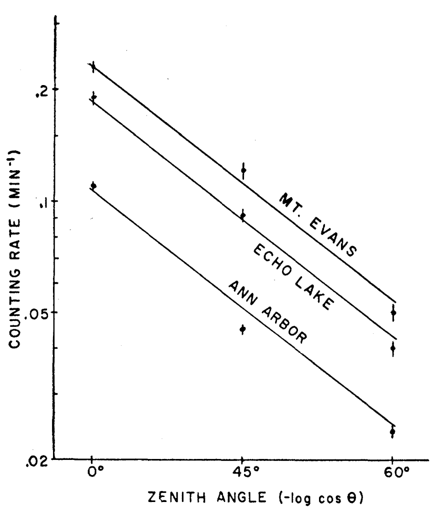

FIG. 2. Zenith-angle dependence of penetrating particles. The straight lines are proportional to $\cos^{2.1}\theta$.

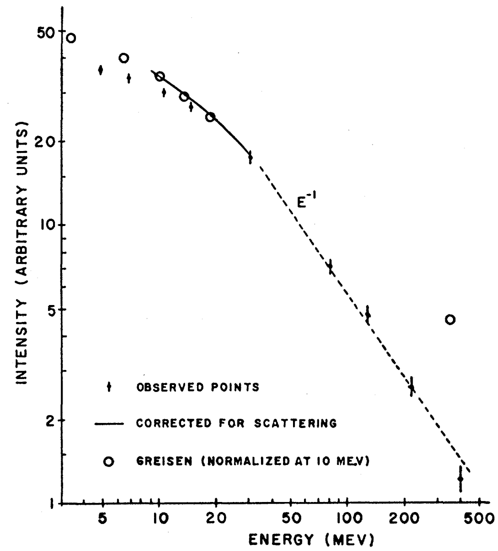

FIG. 3. Integral energy distribution of electrons.

The observed ratio of vertical intensities at Echo Lake and Ann Arbor is 1.72 ± 0.085, and the ratio for Mt. Evans and Echo Lake is 1.22 ± 0.065. Kraushaar found a ratio of 1.71 ± 0.02 for Echo Lake and Ithaca. Rossi *et al.*[17] found 1.22 ± 0.009 for Mt. Evans and Echo Lake.

The good agreement with other observations of penetrating particles indicates that if there are any significant systematic errors they must be of such a nature as to affect only the electrons.

ENERGY DISTRIBUTION OF ELECTRONS

The energy distributions of electrons observed at the various altitudes and zenith angles do not differ by amounts appreciably greater than the statistical errors. Therefore, they can all be represented approximately by the average distribution of Fig. 3, which was obtained by adding together all the electrons of each energy range, exclusive of the Mt. Evans $0_{(1)}$° column. Half of the questionable electrons were counted and the errors indicated are the arithmetical sums of the statistical standard deviations and half the numbers of questionable electrons. The correction for scattering in the central counters was computed in the manner described previously and the corrected distribution is indicated by the solid line extending between 9 and 30 Mev. Between 30 and 400 Mev, the distribution can be represented by E^{-1}. At the high-energy end there is an indication that it is falling off more rapidly, in

[17] Rossi, Hilberry, and Hoag, Phys. Rev. **57**, 461 (1940).

agreement with experiments at higher energies[4,5] which give $E^{-1.5}$. The points marked with circles represent the intensities at low altitude, integrated over all directions, as determined by Greisen.[2] His point at 350 Mev was determined from the electrons penetrating a large thickness of iron, and the energy was calculated by means of shower theory. More recent results indicate that the tails of showers extend over considerably greater distances than was previously thought to be the case,[11,18] and it is probable that this point actually corresponds to an energy much smaller than 350 Mev. The points at low energies were determined with carbon absorbers and should be quite accurate.

A measurement with sufficiently small statistical errors would undoubtedly reveal differences between the energy distributions at different altitudes and zenith angles. In particular, it may be expected that at large zenith angles the number of low-energy electrons is relatively larger than near the vertical, because of scattering in the air. A slight indication of this effect was found, but it is hardly significant statistically.

INTENSITIES OF ELECTRONS

Zenith-Angle Variation

The ratios of electrons to penetrating particles are given in Fig. 4 for electrons with $E>10$ Mev and $E>80$ Mev. The numbers for $E>10$ Mev were obtained by multiplying the number penetrating the second plate by 34/30, in agreement with the corrected curve of Fig. 3. At Mt. Evans, the zenith-angle dependence of the ratios can be represented by $\cos^{0.9}\theta$ for $E>80$ Mev, and by $\cos^{0.7}\theta$ for $E>10$ Mev, giving $\cos^{3.0}\theta$ and $\cos^{2.8}\theta$ for the electron intensities. At Echo Lake, the angular dependence is not very different from that at Mt. Evans, but at Ann Arbor the ratios are approximately independent of the zenith angle, and the electron intensities can be represented roughly by $\cos^{2.1}\theta$ for $E>10$ Mev and $\cos^{2.3}\theta$ for $E>80$ Mev.

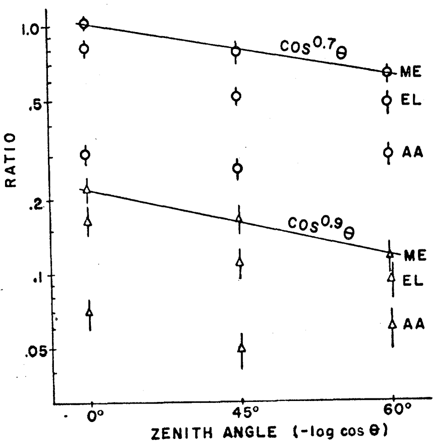

FIG. 4. Ratio of electrons to penetrating particles as a function of zenith angle at Ann Arbor, Echo Lake, and Mt. Evans. The circles represent electrons with $E>10$ Mev and the triangles represent electrons with $E>80$ Mev.

It should be noted that the limits of error indicated in Fig. 4 represent extremely generous estimates when the angular dependence is under consideration. Since the particles of uncertain identification occur in a similar way at all angles, the fraction of them which should be counted is surely about the same in each case. Neglecting this effect, we find the maximum statistical error in the exponent of $\cos\theta$ to be about 0.4.

Experiments using counter telescopes have given much steeper distributions. In particular, Palmatier[3] found $\cos^{3.8}\theta$ for electrons of $E>30$ Mev at Ithaca and $\cos^{4.7}\theta$ at Echo Lake. It is to be expected that the angular distribution will be slightly steeper at higher energies due to smaller scattering in the air, and thus our data for $E>80$ Mev are in strong disagreement with Palmatier's results.

A recent experiment using random expansions of a cloud chamber at 3500 m altitude[4] indicates that both electrons of $E>300$ Mev and penetrating particles have angular distributions of $\cos^2\theta$. However, the statistical uncertainties are large and the data are consistent with a somewhat steeper distribution for the electrons as suggested by an extrapolation of our results to higher energies.

Vertical Intensity

Taking the vertical intensity of penetrating particles as 0.0090 cm^{-2} $sterad^{-1}$ sec^{-1} at Ann Arbor, we find that the vertical intensity of electrons with $E>10$ Mev is 0.0028 ± 0.00025 cm^{-2} $sterad^{-1}$ sec^{-1}. The curve deduced by Rossi[19] for the vertical intensities of electrons gives 0.0022 at the altitude of Ann Arbor. Rossi assigns a lower limit of 10 Mev to these electrons, but it is probable that, due to scattering, not all electrons above this energy were included. The vertical intensity found by Palmatier[3] at Ithaca is 0.0031 for $E>13$ Mev. For the ratio of vertical intensities at Mt. Evans and Ann Arbor Rossi's curve gives 9.5, while we find only 7.1 ± 0.8. For the ratio of vertical intensities at Echo Lake and Ann Arbor, Rossi's curve gives 5.2, Palmatier's results give 5.0, and we find 4.6 ± 0.55.

Integrated Intensity

Assuming an angular distribution of $\cos^{2.1}\theta$, in agreement with Fig. 4, and the vertical intensity given in the preceding paragraph, we find an integrated intensity of 0.0056 ± 0.0005 cm^{-2} sec^{-1} for electrons of $E>10$ Mev at Ann Arbor. Kraushaar[15] has deduced a value of 0.0060 cm^{-2} sec^{-1} for the same quantity at Ithaca, using the results of a measurement by Greisen.[2]

ELECTRONS FROM μ MESONS

A numerical calculation was made of the vertical intensity at Ann Arbor of electrons with $E>10$ Mev

[18] I. B. Bernstein, Phys. Rev. 80, 995 (1950).

[19] B. Rossi, Revs. Modern Phys. 20, 537 (1948).

originating from the decay of μ mesons. The energy distribution of the decay electron in the rest system and the altitude variation of the shape of the meson spectrum were taken into account. The track-length calculations of Rossi and Klapman[20] were used. The greatest uncertainty in the result appears to be due to insufficient information about the momentum spectrum of mesons as a function of altitude. The meson spectra of Sands[21] were used and values for intermediate atmospheric depths obtained by linear interpolation on a semi-logarithmic plot (equivalent to assuming exponential variation with depth for a given momentum). The ratio of electrons with $E>10$ Mev to mesons with $p>150$ Mev/c was found to be 0.183.

The same quantity was calculated in a simplified way, using the method developed by Rossi *et al.*[20,22] This method assumes that in the rest system of the meson the electron always receives 1/3 of the energy, and that the meson spectrum does not vary with altitude. Considering fast mesons only, it is found that the ratio of electrons to mesons at depth h is given by the expression $(1.69\times10^{-4}/N_h)(N/\rho)_{h-113}$. N is the intensity of fast mesons and ρ is the density of air. N_h is to be evaluated at the point of observation and $(N/\rho)_{h-113}$ at a point 113 g cm^{-2} higher. Taking N and ρ from the curves given by Rossi,[19] we find that at Ann Arbor ($h=1000$ g cm^{-2}) the ratio is 0.188, in good agreement with the value obtained by the more complicated method.

For other altitudes and zenith angles the computation was made only by the simplified method, and the results were multiplied by 183/188. Assuming that the shape of the meson spectrum is the same at all altitudes and zenith angles, the ratio for zenith angle θ can be obtained by letting N represent the vertical intensity of fast mesons and evaluating N/ρ at a point 113 $\cos\theta$ g cm^{-2} above the point of observation.

The number of collision electrons, as calculated by Rossi and Klapman,[20] is 0.064 times the number of mesons with $p>150$ Mev/c. It is reasonable to assume that this ratio is independent of altitude and zenith angle.

An approximate correction for the effect of scattering in the atmosphere was made by assuming that all electrons are deflected by the same angle from the direction of production. This angle, α, was taken as 0.25 radian from the calculations of Roberg and Nordheim[23] for the rms deflection of shower electrons with $E>10$ Mev. Under the assumption that the electrons are produced with an angular distribution of $\cos^2\theta$, it can be proved that the distribution including scattering is $\cos^2\theta[1-\sin^2\alpha(1-\frac{1}{2}\tan^2\theta)]$. The correction factor is 0.938 at 0°, 0.969 at 45°, and 1.031 at 60°. The calculated numbers of electrons from decay and collision, including this correction, are given in Table IV.

TABLE IV. Calculated ratios of intensity of electrons ($E>10$ Mev) from decay and collision of μ-mesons to intensity of μ mesons ($p>150$ Mev/c).

	0°	45°	60°
Ann Arbor	0.232	0.222	0.227
Echo Lake	0.324	0.302	0.302
Mt. Evans	0.369	0.338	0.336

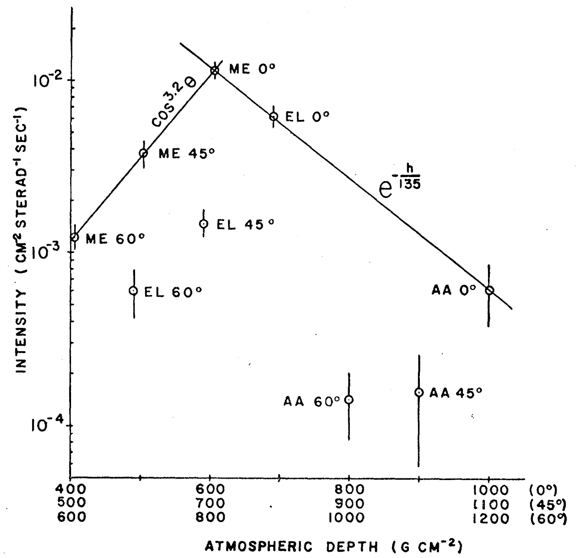

FIG. 5. Intensities of electrons with $E>10$ Mev resulting from nuclear interactions, plotted against atmospheric depth. The points for zenith angle 45° are offset by 100 g cm^{-2} to the left, and the points for 60° are offset by another 100 g cm^{-2}. For a given altitude the graph may be considered as a plot of intensity against $-\log\cos\theta$.

ELECTRONS FROM NUCLEAR INTERACTIONS

The numbers of electrons calculated to originate from μ mesons were subtracted from the observed numbers, and the intensities of the residual electrons are indicated in Fig. 5. To convert the ratios of electrons to mesons into absolute intensities, it was assumed that the intensities of penetrating particles are proportional to $\cos^{2.1}\theta$ and that the intensity at Echo Lake is 1.72 times that at Ann Arbor and at Mt. Evans 1.22 times that at Echo Lake. A correction was made for the protons included in the penetrating particles, taking the proportion of protons as 0.06 at Mt. Evans, 0.04 at Echo Lake, and 0.0075 at Ann Arbor.[24] At large zenith angles the proportion of protons is certainly smaller, but this fact was not taken into account because the correction for protons affects the electron intensity by a maximum of only three percent.

The limits of error in Fig. 5 are the same as those of Fig. 4. The points for 45° and 60° at Ann Arbor are very uncertain because they represent differences be-

[20] B. Rossi and S. J. Klapman, Phys. Rev. **61**, 414 (1942).
[21] M. Sands, Phys. Rev. **77**, 180 (1950).
[22] B. Rossi and K. Greisen, Phys. Rev. **61**, 121 (1942).
[23] J. Roberg and L. W. Nordheim, Phys. Rev. **75**, 444 (1949).
[24] M. C. Mylroi and J. G. Wilson, Proc. Phys. Soc. (London) **A64**, 404 (1951).

tween almost equal quantities and because the calculations of decay electrons at large zenith angles involve considerable approximation.

The variation of vertical intensity with atmospheric depth is represented very well by an expression of the form $e^{-h/L}$, with $L=135$ g cm^{-2}. A rough estimate of the uncertainty in L is ± 15 g cm^{-2}. This altitude dependence is in good agreement with the hypothesis that all the excess electrons originate from nuclear interactions. By way of comparison, Tinlot[25] has found $L=118\pm 2$ g cm^{-2} for the altitude dependence of penetrating showers.

A recent calculation by Stroffolini[26] gives vertical intensities of electrons from μ decay considerably smaller than were used here. The use of his results would give an altitude dependence of excess electrons which is still exponential, but with L about 150 g cm^{-2}.

At Mt. Evans, the zenith-angle dependence of the excess electrons is well represented by $\cos^{3.2}\theta$, and at Echo Lake it is approximately the same. Miller *et al.*[27] found a similar dependence for slow protons at 3300 meters. There is little other evidence available concerning the angular variation of particles associated with nuclear interactions.

Palmatier, following a procedure similar to that used here, subtracted a calculated contribution by electrons due to μ mesons from the measured intensities.[3] He found that the residual intensities, at Ithaca and Echo Lake, are proportional to the expression $e^{-h/160\cos\theta}$. This is a much steeper angular variation than that indicated in Fig. 5. He used a lower energy limit of 30 Mev, but the difference in energy limit should have only a small effect on the zenith-angle variation.

MULTIPLE-ELECTRON EVENTS

In all the preceding analysis, only electrons occurring singly have been included. The numbers given in Table I for electrons occurring multiply are upper limits and must include many which did not pass through the counter telescope. A reasonable guess for the number of allowable electrons capable of penetrating the second plate is half the number listed. This is probably a large overestimate since the angular opening of the telescope is so small. Including half of the listed numbers increases the total vertical intensities by less than 10%. The altitude dependence is not appreciably affected. The change in angular distribution is equivalent to an increase of about 0.1 in the exponent of $\cos\theta$. For the electrons from nuclear interactions, the vertical intensities are increased by 15 or 20% and the exponent of $\cos\theta$ is increased by 0.2.

SUMMARY AND DISCUSSION

Almost all investigations, including the one reported here, give substantial agreement regarding the directional intensities of penetrating particles and their variation with altitude and zenith angle.[1,3,4,15,17]

We find the energy distribution of electrons to be approximately independent of altitude and zenith angle. An average energy spectrum, corrected for scattering in the counters, is presented in Fig. 3. The shape of this spectrum, in the region of low energies, is in good agreement with that determined by Greisen in measurements of the integrated intensity near sea level.[2] There are indications that the points above 50 Mev actually correspond to energies somewhat higher than those given.[12]

We find the zenith-angle dependence of the electron component to be approximately the same as that of the penetrating particles near sea level, and only slightly steeper at mountain altitudes. The observations of Lovati *et al.*[4] with random expansions of a cloud chamber support this conclusion, but Palmatier's counter measurements[3] give a much steeper angular distribution. We have endeavored without success to find a satisfactory explanation for this discrepancy. It is possible that at low energies our results may contain errors due to scattering or to counting electrons which did not pass through the counter telescope, but it is difficult to conceive that such errors could be appreciable at energies above 80 Mev. The only obvious criticism of Palmatier's work is that he does not appear to have made adequate correction for scattering in the central counter. The relation between wall-thickness and spacing of his counters was such that the correction should be almost exactly the same as in the present experiment. However, this correction could have only a negligible effect on the angular distributions.

We find the vertical intensity at Ann Arbor of electrons with $E>10$ Mev to be 0.0028 ± 0.00025 cm^{-2} sterad^{-1} sec^{-1}, in rough agreement with Rossi's data.[19] Palmatier[3] gives a higher vertical intensity, but it is difficult to make exact comparisons because of uncertainties involved in setting the energy limit. Our data give a somewhat slower increase of vertical intensity with altitude than has been indicated by Rossi.

Subtracting the numbers of electrons produced by μ mesons from the observed numbers, we find that the residual electrons have a zenith-angle dependence of about $\cos^{3.2}\theta$ at mountain altitudes. Near sea level, the errors are very large and the angular variation is not well determined. The vertical intensity of these electrons varies exponentially with altitude, with an absorption length of 135 ± 15 g cm^{-2}. This is approximately the form of altitude variation that is to be expected if the electrons originate from nuclear interactions through intermediary steps of very short lifetime, such as the π^0 meson. The expression given by Palmatier[3] for the dependence of the residual electrons on altitude and zenith angle, $e^{-h/L\cos\theta}$, is of the form that would be obtained under the assumption that there are no angular deviations in the production of the electrons or in the

[25] J. Tinlot, Phys. Rev. **74**, 1197 (1948).
[26] R. Stroffolini, Nuovo cimento **10**, 300 (1953).
[27] Miller, Henderson, Garrison, and Sandstrom, Phys. Rev. **85**, 723 (1952).

development of the showers. This assumption is not very realistic and it is to be expected that, because of angular deviations, the zenith-angle distribution will actually be less steep.

The zenith-angle distribution of residual electrons found in the present experiment is much less steep than could be explained under the assumption that the electrons preserve the directions of the primary particles from which they originate. In deriving the numbers of electrons due to μ mesons we found that the scattering of shower electrons has a relatively small effect on the zenith-angle distribution. With a steeper initial distribution the effect is larger, but it does not appear sufficient to explain the small observed zenith-angle variation. The indications are, therefore, that there are appreciable angular deviations in the production and decay processes of the π^0 mesons, and, consequently, that their average energy is not large.

In comparing the results of experiments using counters alone and those using cloud chambers, one might consider the latter as more reliable, in spite of poorer statistics, because each event can be considered individually and the various components can be identified with greater accuracy. In a measurement using only counters it is usual to determine the total counting rate and then subtract the counting rates of all unwanted types of events, and there is always the possibility of overlooking or wrongly estimating some of the extraneous events. We feel that further investigations are required to determine the reasons for the disagreements in the results obtained by the two methods.

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Measurement of the High-Energy End of the Bremsstrahlung Spectrum*†

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Results of a differential measurement of the upper six percent of the bremsstrahlung photon spectrum produced by 500±25 Mev electrons agree with the Bethe-Heitler formula within the combined experimental errors of approximately 25%. The measurement was made by running a collimated beam of 10^3 electrons per pulse through a diffusion cloud chamber and observing the electrons degraded in energy by the production of a gamma ray. Observation of electron-electron scattering events provided a normalization in the center of the energy region investigated. A total of 330 events was observed. The performance of a diffusion cloud chamber under these conditions is discussed.

INTRODUCTION

THE differential cross section for the production of bremsstrahlung by high-energy electrons was first calculated by Bethe and Heitler[1] on the basis of the Born approximation. If one includes the various corrections to this formula—such as the screening effect—the final result should provide an accurate description of the radiation produced by a beam of electrons striking a thin, low-Z target. For a high-Z target, which is commonly used as a radiator, the Born approximation introduces an error.

Experimentally, the cross section has been checked for several primary energies in the medium energy range (10 to 300 Mev). Absolute measurements have been made by observing the initial and final energies of the electron in a cloud chamber, the energy of the gamma ray being determined by subtraction. Curtis[2] found that the shape of the theoretical spectrum agrees quite well with the experimental shape, but that the experimental cross section in lead is about 7% lower than the theoretical value. His primary energy was 60 Mev with a half-width of 10 to 13 Mev in the primary energy spectrum. In the upper 5% of the spectrum he shows one point with an error of ±15%. Fisher[3] did the same sort of experiment using a primary beam whose average energy was 247 Mev; the half-width of his primary energy spectrum was approximately 80 Mev. In the upper 5% of the spectrum Fisher shows two points with approximately 5% errors. He concludes that the experimental cross section is 9% lower than the theoretical.

At this time, investigators are using the photon-

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† This paper is a summary of a thesis submitted in partial fulfillment of the requirements for the Ph.D. degree at Stanford University.

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[1] W. Heitler, *The Quantum Theory of Radiation* (Clarendon Press, Oxford, 1954), third edition.

[2] C. D. Curtis, Phys. Rev. **89**, 123 (1953).

[3] P. C. Fisher, Phys. Rev. **92**, 420 (1953).