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Interpretation of Cosmic-Ray Measurements Far Underground*

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INTRODUCTION AND SUMMARY

MEASUREMENTS of the intensity of cosmic rays as a function of depth underground have shown that the absorption coefficient decreases continually with increasing depth. Experiments at depths between 10 and 70 meters water equivalent (m.w.e.), made with ionization chambers, counters, cloud chambers, and photographic emulsions, have made it clear that the particles detected at these depths are primarily μ-mesons, and that these particles lose energy mainly by ionization processes. Cloud-chamber experiments at still greater depths indicate that the same conclusions can probably be extended at least to a depth of 850 m.w.e. underground. Therefore, the decreasing absorption coefficient bespeaks a continual increase, with increasing depth, of the mean energy of the mesons arriving underground.

Thus, the earth can be used as a filter to absorb the strongly interacting components of cosmic radiation and select μ-mesons having a high average energy. In first approximation, the integral number-energy spectrum of μ-mesons at sea level is a power law, going as E^{-w} with $w \simeq 1.8$ for energies $\gtrsim 10^{10}$ ev; and the rate of energy loss in the ground is a constant, about 2×10^8 ev per m.w.e. To this approximation, the mean energy of the mesons arriving at a depth h (m.w.e.) in the vertical direction is $2\times 10^8 hw/(w-1)$ when they enter the ground and $2\times 10^8 h/(w-1)$ when they reach the depth h. If the conclusions of the first paragraph remain valid, one can therefore do experiments on particles of average energy in the range $10^{11}-10^{12}$ ev by making measurements at depths on the order of 1000 m.w.e. Furthermore, since the mesons must be secondary particles produced by the nuclear-interacting component in the atmosphere, and since the particles causing the interactions must be of much higher energy than the secondaries, one has the possibility of deriving information about nuclear interactions of particles with energy greater than 10^{13} ev from the phenomena associated with the cosmic rays detected underground.

With this in mind, a series of measurements has been made during the last two years at a depth of 1574 m.w.e. (i.e., 1.574×10^5 g/cm^2) in a salt mine near Ithaca. The earth above the salt mine is composed of sedimentary strata, the flatness and uniformity of which are favorable for reliable measurements of intensity at a known depth, and for a measurement of the zenith-angle distribution, since the composition of the ground does not vary with inclination. The position of the salt mine near the center of a broad anticline assured that the strata were almost perfectly level, except at the surface of the ground, where there was a slight inclination. Data on the composition of the strata were supplied by Mr. Samuel Whitman of the Cayuga Rock Salt Company. Most of the ground consists of shales of various types, with average density 2.65 g/cm^3 and average atomic number 11.

The interior of the salt mine was perfectly dry, and constant in temperature the year around. The background counting rate in the counters, which were all shielded with lead, was about 40 times less than at sea level. The low counting rates and the constant temperature favored long life of the counters and reliable counting.

The mine also possessed an auxiliary shaft, through which it was possible to run cables so as to record coincidences between counters on the surface of the ground (elevation 134 m) and those 594 meters underground.

Part of the review below deals with evidence indicating that at 1600 m.w.e. the cosmic radiation is still composed of ionizing particles similar in behavior to μ-mesons, accompanied by a secondary soft component. In Section A an anticoincidence experiment is described, in which the absorption in lead of the penetrating component capable of producing fourfold coincidences was measured and compared with the absorption coefficient of the total intensity in the ground. The agreement found in this comparison indicates that the particles detected are the ones which have penetrated the ground, rather than local secondaries. In Section C the rate of twofold coincidences is compared with the rate of fourfold coincidences; and the agreement indicates that the penetrating particles causing twofold coincidences are ionizing and identical with those producing the fourfold and fivefold coincidences. These two experiments directly contradict the previous evidence suggesting a non-ionizing character of the penetrating radiation.

Further evidence supporting the μ-meson nature of the cosmic rays at 1600 m.w.e. is given in Sections B, E, and G, where the weak nuclear interaction and high average energy of the particles are demonstrated.

Another topic investigated is the mechanism by which the penetrating particles originate. If this mechanism

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is principally the decay of π-mesons, or decay of any meson with similar ratio of mass to lifetime, the penetrating particles of a given energy should not be isotropic, and there should be a positive correlation between intensity and average temperature of the atmosphere. These predictions are developed mathematically in the appendix. In Section B, measurements of intensity as a function of zenith angle are described and compared with the variation of intensity with depth. In Section D, the correlation of intensity with temperature is discussed. Both sets of measurements show evidence for the decay process and are consistent with the predictions made on the assumption that $\pi-\mu$ decay is the only means of creating the penetrating component; but statistical errors and the possible existence of other mesons (such as the κ-meson), which also undergo μ-decay with an appreciable lifetime, limit the definiteness of the conclusions that can be drawn.

In Section E, the coincidences between extensive air showers and cosmic rays 1600 m.w.e. underground are described. From these, the average energy of the primaries responsible for the penetrating component is estimated; and the approximate relation is found between the number of mesons penetrating 1600 m.w.e. and the number of electrons in the air shower, both generated by a single primary. It appears from the data that practically all the mesons at 1600 m.w.e. are associated with air showers containing more than about 400 electrons at sea level.

In Section F, data are presented on the showers of mesons occurring at 1600 m.w.e. underground. Evidence is given that these showers are created in the air. The mean energy of the primaries responsible for the observed meson showers is estimated, both from the decoherence curve of the meson pairs and from the sizes of the associated air showers. This energy is more than 10^{15} ev, much greater than the energy responsible for most of the penetrating particles observed singly. Also derived is the multiplicity distribution of the meson showers and the approximate relation between the multiplicity and the primary energy. The study of meson showers far underground is shown to be analogous to experiments on the core structure of extensive air showers.

In Section G the nuclear interactions produced by the penetrating component are discussed. The evidence is derived from hodoscope records indicating penetrating showers, from an analysis of the intensity-*vs*-depth curve, and from an attempt to detect neutrons of moderate energy emitted in evaporations of the residual excited nuclei. The estimated cross sections for nuclear interaction are greater than the average cross sections of mesons at a depth of 20 m.w.e., but are not too great to be accounted for by electromagnetic (i.e., photonuclear) interactions.

Finally, in Section H the estimates of energy of the primaries causing the single mesons and the meson showers at 1600 m.w.e. are assembled, and the absolute frequencies are calculated. These data provide two new points which are added to previous information on the number-energy spectrum of the primary cosmic rays. Evidence is presented that the primaries of energy around 10^{15} ev are composed mostly of protons and α-particles rather than heavier nuclei.

These experiments were made possible by the generous cooperation of the managers of the Cayuga Rock Salt Company of Myers, New York, and by financial support extended by the Geophysical Research Directorate of the Air Force Cambridge Research Laboratories, AMC. We wish especially to thank Mr. Samuel N. Whitman, engineer of the mine, and Mr. C. F. Van Amber, foreman of the Machine Shop in the Laboratory of Nuclear Studies, for their assistance.

A. LOCAL ABSORPTION OF THE PENETRATING IONIZING PARTICLES

There has been fairly general agreement that the cosmic-ray particles which penetrate to great depths underground are μ-mesons. This belief is in agreement with a reasonable extrapolation of the known composition, energy spectrum and behavior of the cosmic rays observed near sea level; and there is no known particle, charged or neutral, other than the μ-meson, which has a penetration probability resembling that necessary to explain the experimental data on intensity *versus* depth underground. However, the belief can be put to experimental test in several ways, one of which is by measurement of the local absorption of the ionizing particles.

Detectors placed underground may presumably respond both to the primary component (not meaning primary cosmic rays, but the component created in the atmosphere which penetrates to the great depths underground) and to secondaries generated near the detecting apparatus by the primary component. When the depth underground is increased by an amount Δh, the frequency of detected primary rays goes down in proportion to the absorption of the primaries in Δh; but the frequency of detected secondary particles only decreases in proportion to the difference between the absorption and the production of secondaries in Δh. Therefore the measured absorption indicates chiefly the absorption of the primary particles. However, if an absorber of thickness Δh is placed within a cloud chamber or between the counters of a counter telescope, the total absorption of the primaries and the secondaries may be observable. The difference between the absorption in matter placed between the counters and the absorption produced by increasing the depth underground has therefore frequently been used as a measure of the rate of production of secondaries underground by the primary component.

For instance, Clay and van Gemert[1] by such means studied the production of secondaries at great depths.

[1] J. Clay and A. van Gemert, Physica **6**, 497 (1939); J. Clay, Revs. Modern Phys. **11**, 128 (1939).

Tiffany and Hazen[2] observed both production and absorption of secondaries in lead plates within a cloud chamber, and identified the secondaries as electrons. Randall and Hazen[3] measured the absorption with counters, and identified the secondaries as electrons by comparing the absorption in carbon with that in thin layers of lead.

Experiments involving twofold coincidences between Geiger counters separated by very thin absorbers[4–7] have demonstrated the existence of a second type of soft radiation. By its directional distribution, its erratic variation with depth, its strong absorption in a few millimeters of lead and its low efficiency for discharging a Geiger counter, this radiation has been indentified with gamma-rays from radioactive materials present in the earth. The gamma-rays cause almost all the counting rate in single counters, and, apparently by double Compton effect, cause a large fraction of the twofold coincidence rate between counters separated by no absorber except the counter walls. However, the efficiency of the scattered gamma-rays is so low that the number of threefold coincidences produced by them is completely negligible.

The presence of ionizing secondaries more penetrating than electrons may be searched for by measuring the absorption in thicknesses of lead greater than the range of secondary electrons. If the absorption is found to be no greater than that produced by an equivalent increase in depth underground, the ionizing particles of greater range than electrons must not be secondaries, but must belong to the primary component. Thicknesses of lead and earth taken to be equivalent in such a test should be equal with regard to absorption of ionizing particles, particularly mesons, near the end of their range. Equivalence with regard to radiative losses or other processes important only at high energy is not essential, since such processes do not stop the particles.

Previous measurements of this type, made at depths beyond the so-called "knee" of the intensity-depth curve (around 300 m.w.e.), have not had great statistical accuracy, and have not all been in agreement with each other. The absorption curves reported by Barnóthy and Forró at 1000 m.w.e.[4] and by Miyazaki at 3000 m.w.e.[8] suggest the existence of numerous secondary ionizing particles of greater range than electrons; while several other experiments at depths from 300 to 1400 m.w.e.[1,2,9–11] indicate local absorptions small enough, within the statistical errors, to be consistent with the nonexistence of such secondaries. Probably the most conclusive of these experiments is the cloud-chamber study by Tiffany and Hazen,[2] because they observed the penetrations and stoppings directly, not by inference from the differences of counting rates measured at different times with different amounts of material around the detector. In 2-in. Pb they observed only 2 stoppings among 454 penetrating particles. This is consistent with the mean number of stoppings expected, 0.63, if these particles constitute the primary component; but it does not preclude the existence of fairly numerous penetrating secondaries.

The present absorption experiment was undertaken in the hope of improving on the statistical accuracy or reliability of the above-mentioned measurements, especially avoiding the following hazards known to be present in counter measurements:

(1) False indication as to direction of the path of a penetrating particle is frequently given when one or more of the counters in telescope arrangement is struck by secondary electrons accompanying the penetrating particle. Thus an apparent absorption of a penetrating particle may occur when only a secondary electron has been stopped. This effect is particularly serious, compared with the number of single penetrating particles recorded, when a telescope of narrow angular aperture is used.

(2) Particles may be scattered out of a telescope, particularly when it is narrow, with frequency comparable to that of particles stopping in the absorber.

(3) Unexpected and complex effects may occur upon change of amount and position of absorber. Time variation of the intensity may also make small differences between rates measured at different times unreliable. Hence it is desirable to measure the number of stoppings directly instead of as the difference between rates obtained with different absorber geometries.

(4) If the absorption is as small as expected for μ-mesons, it is almost impossible to obtain statistical significance in the absorption by comparing coincidence rates measured independently.

(5) Geometrical coherence between primary and secondary particles may conceal absorption of the secondaries, especially if broad counter trays are used with no resolution within the trays, and especially if the roof above the apparatus is low. This effect is insignificant if the primary component is neutral, but it has in some cases, for instance, resulted in an underestimate of the number of electrons accompanying the charged penetrating particles.

These dangers were minimized by measuring the absorption with anticoincidences instead of by differences between coincidence rates, by providing plentiful coverage with the anticoincidence counters, by using broad counter trays, and by using crossed trays and hodoscope recording of the pulses in all of the counters in order to provide a maximum of resolution in both of the horizontal coordinates.

A diagram of the apparatus is given in Fig. 1. The counter trays B and D were at right angles to the other trays. Mixing circuits connected to trays A, B, C, and D generated a master pulse whenever one or more counters was discharged in each of these trays. The master pulse

[2] O. L. Tiffany and W. E. Hazen, Phys. Rev. **77**, 849 (1950).
[3] C. A. Randall and W. E. Hazen, Phys. Rev. **81**, 144 (1951).
[4] J. Barnóthy and M. Forró, Z. Physik **104**, 744 (1937); Phys. Rev. **55**, 870 (1939); Phys. Rev. **58**, 844 (1940); Phys. Rev. **74**, 1300 (1948).
[5] Miesowicz, Jurkiewicz, and Massalski, Phys. Rev. **77**, 380 (1950).
[6] J. Gierula, Acta Phys. Polonica **11**, 36 (1951).
[7] L. M. Bollinger, Cornell University thesis (1951); Phys. Rev. **79**, 207 (1950); Phys. Rev. (to be published).
[8] Y. Miyazaki, Phys. Rev. **76**, 1733 (1949).
[9] Nishina, Sekido, Miyazaki, and Masuda, Phys. Rev. **59**, 401 (1941).
[10] V. C. Wilson, Phys. Rev. **55**, 6 (1939); V. C. Wilson and D. J. Hughes, Phys. Rev. **63**, 161 (1943).
[11] Randall, Sherman, and Hazen, Phys. Rev. **79**, 905 (1950).

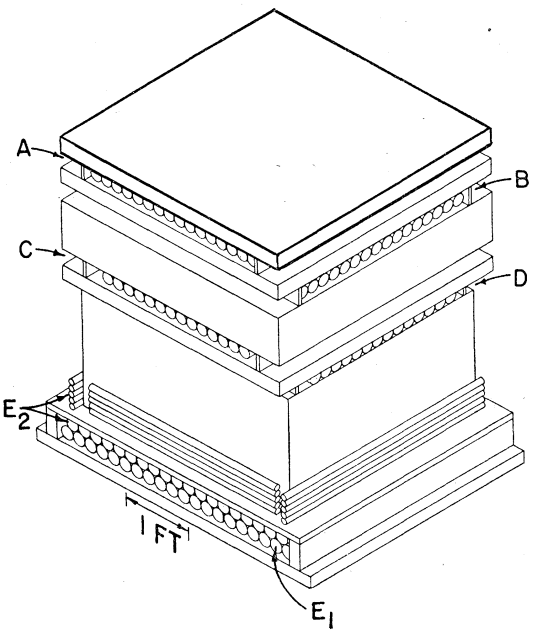

FIG. 1. Arrangement of counters and absorbers used to measure the absorption of the penetrating particles, and the intensity as a function of zenith angle; also to observe the penetrating showers generated underground, and the mesons coincident with extensive air showers.

was put in coincidence with each counter in these trays and in tray E_1, and a photographic record was obtained showing which counters had been discharged. Coincident pulses in tray E_2 were similarly registered, except that for economy of tubes these counters were grouped into only 13 independent channels.

The coincidence resolving time was 30–40 μsec and the counting rate in individual counters about 0.3 per second. Therefore chance coincidences were completely negligible.

The layers of lead and iron above the top tray and between the first four trays served to absorb electrons incident from the roof or generated in the absorbers and counter walls. Electrons generated in a layer immediately above any counter tray would usually remain close to the path of the penetrating particle, thus minimizing the errors in the inferred path. The lead under the bottom tray reduced the background counting rate due to gamma-rays from the rock, and protected against electrons scattered up from the floor. The lead between trays B and C was usually 8 in. thick; for a short time it was only 4 in. thick, but no differences were detectable in the rates or types of events recorded when the thickness was changed. The thickness of lead between trays D and E was varied in the course of the experiment.

Thus, in order to generate a master pulse, it was required that a particle traverse about 12-in. Pb+1.5-in. Fe between trays A and D, in addition to the counter walls. The hodoscope records could then be examined to see whether or not the particles also penetrated the additional absorber between tray D and tray E, discharging counters in trays E_1 and E_2.

Some of the photographs showed complex events in which numerous counters were discharged in individual trays and one could not be sure of the direction of any single particle. These events could not be used for the absorption measurement, though they are discussed below in connection with other problems. However, most of the photographs showed simple events in which only one counter (or occasionally two adjacent counters) was struck in each of trays A, B, C, and D. In these cases the direction of the path could be determined within about seven degrees and the lateral position within about an inch. The arrangement of trays at right angles is responsible for this resolution, and is equivalent to having about 50,000 narrow telescopes operating simultaneously.

From the events in which the direction was clear, those were selected which indicated paths that should certainly cross the counter trays E_1 and E_2, even allowing for several inches sideways scattering of the particles in crossing the absorber. Figure 2 exhibits the differential aperture‡ of the apparatus, defined by trays A, B, C, and D, as a function of zenith angle; the figure also shows the aperture *vs* zenith angle for those telescopes permitted in the absorption measurement.

The selected events were classified according to whether or not a counter in trays E_1 or E_2 was discharged, with the results shown in Table I. In order to eliminate spurious effects, if any, due to counter ineffi-

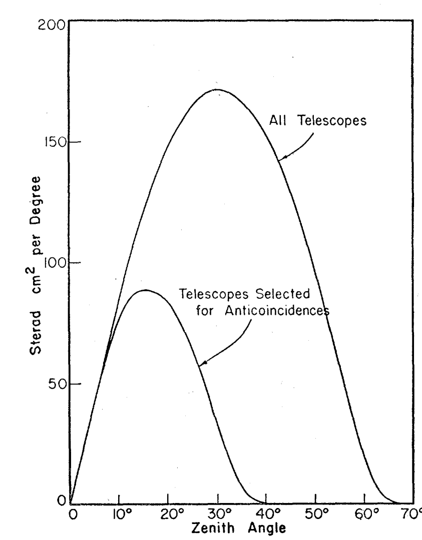

FIG. 2. Differential sensitivity of the apparatus shown in Fig. 1, as a function of zenith angle. "All telescopes" refers to all directions which cross counters in the four trays A, B, C, and D. "Telescopes selected for anticoincidences" refers to those combinations of counters in trays A to D which define lines that traverse tray E_1 farther than 3 inches from the edge.

‡ By differential aperture is meant $Ad\Omega/d\theta$, with A representing sensitive area normal to the paths of the particles, averaged over all azimuths, and $d\Omega$ the solid angle included within an interval $d\theta$ of zenith angle.

TABLE I. Particles stopping in absorbers.

Absorber between D and E	Particles within required directions	Number apparently stopping	Percentage absorbed
1-in. Fe+counter walls	932	2	0.2
1-in. Fe, counter walls, and 2-in. Pb	1910	5	0.3
(Total with small amount of absorber)	2842	7	0.25 ± 0.09
1-in. Fe, counter walls, and 20-in. Pb	4579	57	1.25 ± 0.17

ciency or other causes, the experiment was run with thin as well as thick absorbers between trays D and E, but never with less than 1-in. Fe in this position, which should be sufficient to absorb most scattered electrons. The two series of measurements with thin absorbers are not significantly different; therefore the average of these data was considered as representing the apparent absorption in the average absorber thickness, 1-in. Fe+1.35-in. Pb. This absorption was subtracted from the apparent absorption in 1-in. Fe+20-in. Pb to obtain the net absorption, 1.00 ± 0.19 percent, in 18.65-in. Pb. Comparison of this result with the thin absorber data indicates an apparent "background" of 0.12 ± 0.10 percent, which is hardly significant.

The results given in Table I are in agreement with the results given in references 1, 2, 9–11, but are in complete disagreement with those of references 4 and 5, according to which we should have observed thousands of particles stopping instead of about 60. The fact that we obtained reasonably consistent results with both thin and thick absorbers between D and E, and the fact that changing the thickness between trays B and C by 4-in. Pb produced no noticeable change in the coincidence or anticoincidence rates, show that the small measured absorption was not due to the chance selection of two total thicknesses corresponding to maxima in a complex absorption curve, but that the slope of the absorption curve is small at all thicknesses beyond 8-in. Pb. We must conclude that Barnóthy and Forró and Miyazaki were misled by errors of the types discussed above, to some of which their rather narrow telescopes should have been particularly sensitive.

In Table II the local absorption in lead is compared

TABLE II. Absorption in lead and earth.

Change in apparent absorption due to change in absorber thickness	1.00 ± 0.19 percent
Increase in absorber thickness	522 g/cm^2 Pb
Equivalent thickness of earth for stopping mesons near the end of their range (neglecting scattering)	343 g/cm^2
Equivalent thickness of earth corrected for relative scattering of mesons in Pb and earth near end of range	363 g/cm^2
Absorption in equivalent earth (from data of reference 7): $(3.5\pm0.3)\times\frac{363}{157{,}400}\times100$	0.81 ± 0.07 percent

with the absorption in earth outside the telescope. For this comparison, the range-energy relations of μ-mesons were used, with a small correction[12] for the difference in scattering of mesons in the different materials. The absorption in earth was calculated by using the usual power law representation of the intensity-*vs*-depth relation, and using for the exponent, $m=-(h/I)(dI/dh)$, the value 3.5 ± 0.3, recently measured by Bollinger[7] between 1500 and 1840 m.w.e.

Within the statistical errors, it is found that the absorption in lead within the telescope equals the absorption of the primary component in a thickness of earth which would be equivalent to the lead for μ-mesons. Therefore the ionizing particles capable of penetrating 12-in. Pb are all, or almost all, part of the primary component that has been created in the air and has traversed all the earth above the apparatus.

The following reservation must accompany this conclusion. If the primary component is charged, but accompanied by secondaries capable of penetrating 12-in. Pb, our experiment would only reveal the absorption of those secondaries with paths separated from the primaries by more than about 20 inches. This is because we rejected events in which the direction was made too ambiguous by the discharge of multiple counters in the upper trays.

Subject to this reservation, the data can be used to set an upper limit on the production of penetrating secondaries by the primary component, which from now on we shall refer to as μ-mesons. That is, a limit can be given to the production of secondaries of range exceeding 12-in. Pb and with paths which extend, before absorption of the secondaries, to distances of at least 20 inches from the path of the primary (part of which separation could occur while traversing the 40-in. air path above the top counter tray). As explained above, the local production of secondaries is equal to the difference between the measured absorptions in matter within and exterior to the apparatus. If one allows statistical fluctuations up to twice the standard error, the data set an upper limit to the excess absorption, equal to 0.6 percent of the number of μ-mesons traversing the apparatus. Assuming that the secondaries are stopped by ionization, this upper limit refers to the production of secondaries in 363 g/cm^2 of earth ($Z=11$). If the multiplicity of production resulting (possibly via a cascade process) from a single interaction of a μ-meson is ν, the upper limit to the cross section is

$$\sigma_{\max}=\frac{6\times10^{-3}}{\nu\times363\times6\times10^{23}}=\frac{3\times10^{-29}}{\nu}\text{ cm}^2\text{ per nucleon.}$$

Probable values of the cross section are of course much lower than this value.

[12] H. P. Koenig, Phys. Rev. **69**, 590 (1946); W. L. Kraushaar, Phys. Rev. **76**, 1045 (1949). At energies beyond a few times μc^2, the correction for scattering is found to decrease in proportion to $(1/E)\log(1+E/\mu c^2)$.

This observation is of interest in connection with the recent report by Braddick, Nash, and Wolfendale[13] of evidence for an elementary process whereby μ-mesons produce, with a large cross section, penetrating particles that also resemble μ-mesons. Our data tend to contradict this: at least they indicate, at a higher meson energy, a lower cross section than that given by Braddick *et al.*, provided the angles of production and the ranges of the secondaries are great enough for them to be more than 20 inches from the paths of the primaries.

Even at a depth of 40 m.w.e. below ground (about twice the depth of Braddick's observations), an experiment by Amaldi, Castagnoli, Gigli, and Sciuti[14] has shown that production of penetrating secondaries by μ-mesons occurs with a cross section at least ten times smaller than that deduced by Braddick *et al.*

B. ABSOLUTE VERTICAL INTENSITY AND ZENITH-ANGLE DEPENDENCE

The intensity of penetrating particles as a function of zenith angle was derivable from the hodoscope photographs described above, taken with the apparatus drawn in Fig. 1. Each picture showed the inclination of the trajectory in two projections, one defined by the counters struck in trays A and C, the other by the counters discharged in trays B and D. The counters hit in trays E_1 and E_2, or the failure to strike these trays, provided a check, sometimes accurate and sometimes crude, on the angles indicated by trays A and C or B and D.

The differential sensitivity *versus* zenith angle of the telescopes defined by trays A, B, C, and D is shown in the upper curve of Fig. 2. The absorbers between trays A and D set a minimum range for detection equal to 12-in. Pb+1.5-in. Fe+counter walls. The corresponding lower energy limit for μ-mesons is 500 Mev.

We believe that the presence of the absorbers and the use of broad counter trays, with hodoscope recording to afford maximum resolution, were essential features of our experiment; and that with simpler coincidence arrangements, especially with a single telescope of narrow aperture, the numerous secondary electrons far underground must cause large errors in the measured intensity and its zenith-angle variation.

For instance, at 730 m.w.e. Barnóthy and Forró[4] measured the threefold coincidence rate of a narrow counter telescope with no absorber, as a function of zenith angle θ, and corrected roughly for the electron showers by subtracting the rate recorded with the telescope horizontal. Their results approximated a power law, $\cos^n\theta$, with $n \simeq 1.7$. At a similar depth, 850 m.w.e., Randall and Hazen[3] obtained a quite different exponent, $n = 2.8 \pm 0.1$, by measuring the coincidence rate between two horizontal trays separated by 15-cm Pb, as a function of the vertical distance between them. In both experiments, the effect of secondary electrons would be to reduce the variation of counting rate with change of the disposition; but with the method of Barnóthy and Forró, this would make the intensity seem too uniform, while with the method of Randall and Hazen, it would make the intensity seem too concentrated in the vertical direction. The discrepancy is therefore to be expected, and the correct exponent at about 800 m.w.e. must lie between the two values reported.

With our apparatus, the most frequent type of event recorded (50 percent of all pictures) showed only one counter struck in each tray, and the counters hit in trays E_1 and E_2 rarely failed to be in line with the counters struck in A and C. Similarly, when the counters in A and C or B and D indicated a line that should miss trays E_1 and E_2, usually no counter in those trays was discharged. Usually, therefore, the zenith angle was not much in doubt. The resolution in projected angles was $\pm 7°$ near the vertical direction, and improved with increasing inclination.

Another frequent event (16 percent of the pictures) showed single counters struck in all but one or two trays, in which two adjacent counters were discharged, the path being sufficiently inclined that a single particle could have traversed both counters. We interpreted these events as being caused by single particles. Under this assumption the angular resolution was even better than when single counters were discharged in all the trays. In some cases, of course, this assumption was wrong, but the errors introduced were not large and were almost equally frequent in opposite directions; thus they had little effect on the angular distribution.

Twenty-six percent of the pictures showed clear evidence of secondaries in one and only one counter tray. In about half of these cases, because of the help given by trays E_1 and E_2, there were still two trays to define each coordinate of the direction without error due to secondaries. In most of the cases, furthermore, the number of extra counters discharged was small, and these counters were adjacent, so the uncertainty in angle was not very large. A first-order cancellation of errors was made by classifying the events according to the mean angle indicated by the discharged counters.

The remaining, more complex pictures, 8 percent of the total, showed more than one particle in 2 or more trays. Some of these events contained well-defined shower cores or parallel penetrating particles and hence could be classified as to zenith angle, occasionally with great accuracy. In most of the events, the close grouping of the counters that were hit and the extra information from trays E_1 and E_2 allowed classification in angle without much uncertainty.

As in cloud-chamber experiments, the many information channels of our apparatus made us aware that the counting and classifying of events were imperfect, but

[13] Braddick, Nash, and Wolfendale, Phil. Mag. **42**, 1277 (1951); H. J. J. Braddick and G. S. Hensby, Nature **144**, 1012 (1939).
[14] Amaldi, Castagnoli, Gigli, and Sciuti, Nuovo cimento **9**, 453 (1952).

they also provided assurance that gross misinterpretations would be rare.

Thus of 18,499 pictures examined in this study, only two percent could not be classified as to zenith angle. It was thought desirable to keep this number low, even at the expense of a few errors in the angles of classified events, because of the variation in sensitivity of the apparatus to showers arriving at different angles. To compensate for not classifying two percent of the pictures a correction of two percent was applied to the running time. Another two percent reduction in running time had to be made because of an artificial insensitivity introduced for 15 seconds after each recorded event, while the camera film was advanced.

For each of the possible combinations of counters that might be struck by a single particle crossing trays A, B, C, and D in a straight line, the zenith angle and the aperture times sensitive area were calculated. Account was taken of the cylindrical shapes of the counters and the insensitive spaces occupied by counter walls. Except at large projected angles, the apertures and sensitive areas were limited in both directions by cylindrical counter walls and were thus independent of the effective lengths of the counters. At projected angles beyond 45° this was no longer true, thus the calculated apertures contain an uncertainty that increases with zenith angle. An estimate of this uncertainty has been included in the calculation of experimental errors.

The number of telescopes at different angles was too great to allow statistical accuracy in the rates of single telescopes; hence those at nearly equal zenith angles were grouped together. Weighted average values of the cosine of the zenith angle θ were computed for each group of telescopes. Figure 3 contains a logarithmic plot of the intensities *versus* $\cos\theta$, with standard errors indicated.

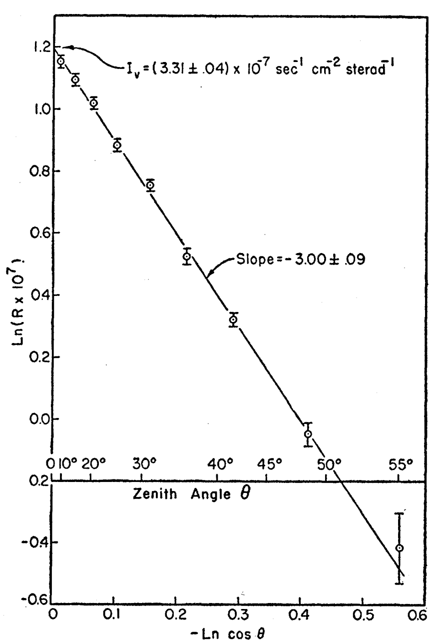

FIG. 3. Intensity *vs* zenith angle of the penetrating particles detected at 1574 m.w.e. A further correction for the effect of electron secondaries reduces the vertical intensity I_v by 2 percent and increases the exponent n by 2 percent.

The above treatment of the data automatically averaged the rates over azimuth at each zenith angle. This provided a first-order correction for the fact that the surface of the ground over the mine was inclined. The second-order corrections for the slope of the ground were calculated, and although they were not large enough to be important, they were applied as corrections to the mean values of $\cos\theta$ before drawing the graph in Fig. 3.

The points in Fig. 3 agree well with a straight line, expressing a power law between intensity I, and cosine of the zenith angle θ. The relation

$$I = I_v \cos^n\theta$$

was assumed and the method of least squares applied to determine the values of the constants I_v and n, with the following results:

$$I_v = (3.31 \pm 0.04) \times 10^{-7}\ \text{cm}^{-2}\ \text{sec}^{-1}\ \text{sterad}^{-1},$$
$$n = 3.00 \pm 0.09.$$

The errors quoted include rough estimates of the systematic errors considered thus far, as well as statistical errors.

The above results still contain a small error due to secondary electrons. One-third of the pictures showed the presence of secondaries that discharged one or more counters in addition to the ones discharged by the meson. In such events, if the path of the meson missed one of the trays by a small distance, a secondary could strike that tray and cause a coincidence to be recorded, contributing to an apparent intensity that is too high. Such events could only occur with appreciable probability when the meson struck 3 of the 4 required trays and passed within about 2 in. of the edge of the fourth tray. Therefore, because of the large dimensions of the trays, the error is only about 2 percent near the vertical direction. The effective area decreases with increasing zenith angle; hence the error increases with zenith angle and is about 4 percent around $\theta = 45°$. These estimates are considered to be accurate to within 50 percent. The corrections lead to the following values of I_v and n:

$$I_v^* = (3.25 \pm 0.05) \times 10^{-7}\ \text{cm}^{-2}\ \text{sec}^{-1}\ \text{sterad}^{-1},$$
$$n^* = 3.06 \pm 0.10.$$

Corrections for the effect of secondaries, usually larger in magnitude, should have been applied in all previous measurements of underground intensity, except those of Bollinger[7] in which the shielding and the symmetry make the errors negligible.

Our results may be compared with those of Bollinger,[7] obtained with a quite different counter arrangement, but also with hodoscope recording and with adequate shielding against secondary electrons. For absolute vertical intensities he obtained $I_v = 3.90 \times 10^{-7}$ at 1500 m.w.e. and 1.91×10^{-7} at 1840 m.w.e., with statistical errors of about 2 percent in I_v, and uncertainties of about one percent in the depths. From these values one computes $3.30 \pm 0.10 \times 10^{-7}$ for the expected intensity at 1574 m.w.e. where the present measurement was made. For the zenith-angle variation at angles less than 45°, he found a cosine power law with exponent $n = 2.90 \pm 0.19$ at 1500 m.w.e. and 3.13 ± 0.17 at 1840 m.w.e., leading one to expect $n = 2.97 \pm 0.15$ at 1574 m.w.e. The agreement with the present measurement is good both in absolute intensity and in shape of the distribution.

Interpretation of the Results

The reason for concern about the precise value of n is the implication of this value with regard to the nature and origin of the particles which penetrate far underground. Suppose that these particles were produced directly in nuclear interactions of the primary cosmic rays, and that the particles did not decay; also that the secondaries retain the direction of motion of the primaries—an assumption that is justified at energies large compared with 2×10^9 ev. Then, since the primaries are isotropic, the secondaries of a given energy would be isotropic and the intensity would depend only on the amount of matter traversed. If I represents intensity of the secondaries, h represents vertical depth in g/cm^2 measured from the top of the atmosphere, and θ represents zenith angle, I would be a function of $(h/\cos\theta)$. Defining

$$m = -(h/I)(\partial I/\partial h) \text{ and } n = (\cos\theta/I)(\partial I/\partial \cos\theta),$$

m and n would be equal if evaluated at equal $h/\cos\theta$.

If the secondaries decay appreciably, the loss by decay would increase with θ, because the average density of the atmosphere where the secondaries are produced decreases with increasing θ, and also because the distance to the ground from a given pressure level increases with θ. The result would be to make the intensity at a vertical depth $h/\cos\theta$ greater than the intensity at a vertical depth h in a direction θ, in spite of the fact that equal amounts of matter lie along both paths. In other words, n would exceed m.

Suppose however that the particles observed underground are not directly produced, but result from decay of secondaries produced by the primaries. Then, increasing inclination will favor the production of the particles observed underground, and m will exceed n.

The most likely situation is that the particles observed underground are μ-mesons, produced by decay of π- or κ-mesons or both, these being produced directly by the primaries. In that case the difference $m-n$ can be represented by the difference of two terms, $\delta - \delta'$, where δ is the positive effect due to decay of the parents, and δ' is the negative effect due to decay of the μ-mesons. If more than one type of decay process contributes to the μ-meson intensity, $\delta = \sum f_i \delta_i$ where f_i is the fraction of the μ-meson intensity due to the i^{th} type of decay process and δ_i is the value of δ associated with this process alone.

The values of f_i, δ_i, and δ' depend on the decay probabilities, which in turn depend not only on the mean lives of the particles involved, but also on the energies. Therefore the relative contributions of the different decay processes to the difference $m-n$, and the value of this difference as well, should vary with the depth underground at which the measurements are made. An experimental measurement of $m-n$ at a given depth, or preferably as a function of depth, would thus offer a means of testing various hypotheses as to the mechanism of the origin of the particles observed underground.

Equations representing the effects described above are derived in the Appendix. Figure 4 shows the expected difference between m and n as a function of depth, under the assumption that the particles detected underground are μ-mesons created by decay in the atmosphere, and under three alternative hypotheses: production entirely by $\pi-\mu$ decay, production entirely by $\kappa-\mu$ decay, and production resulting from equal numbers of π- and κ-mesons. (The tentative estimates of the parameters of the $\kappa-\mu$ decay process, based on the observations of O'Cealleigh,[15] may be considerably in error.) At the depth of the present experiments, the value of δ' (owing to $\mu-e$ decay) is practically zero, the value of δ associated with $\pi-\mu$ decay is about 0.9, near its maximum value 1, while the value of δ associated with $\kappa-\mu$ decay is probably small compared with 1. Therefore an accurate determination of m and n might make it possible to distinguish between the hypotheses suggested.

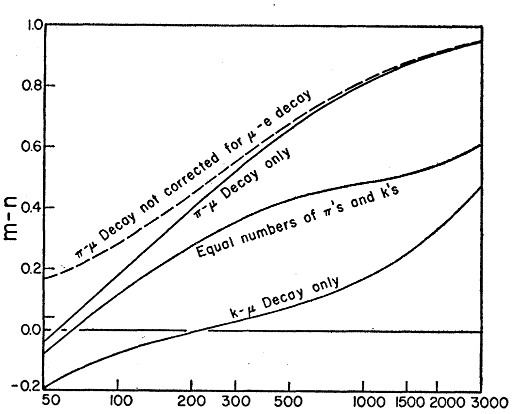

FIG. 4. Calculated difference between the exponent m in the depth dependence of the intensity and the exponent n in the zenith angle distribution, under various hypotheses as to the nature of the parents of the μ-mesons. The solid curves take into account the effect of $\mu-e$ decay, while the single dashed curve illustrates the magnitude of this correction. The calculations are explained in the Appendix.

[15] C. O'Ceallaigh, Phil. Mag. **42**, 1032 (1951); Daniel, Davies, Mulvey, and Perkins, Phil. Mag. **43**, 753 (1952).

By intensity measurements at 1500 and 1840 m.w.e., Bollinger[7] has found the average value of m in this depth interval to be 3.5, with a standard statistical error of ± 0.15 and an uncertainty of ± 0.2 due to possible error in the relative depths. In order to combine this measurement with those of other authors and to estimate the variation of m with depth, intensity *vs* depth measurements from several sources have been combined in Fig. 5, with the following normalization procedures and adjustments:

(a) The errors where shown are standard statistical errors. Because of logarithmic scale, in many cases these were too small to appear outside the circles. No attempt was made to indicate possible systematic errors.

(b) The depths are expressed in hundreds of g/cm² of earth, calling 100 g/cm² one m.w.e., without correcting for the differences in energy loss between the various kinds of earth or between earth and water. Ten m.w.e. was included in the depth to represent the equivalent thickness of the atmosphere. None of the measurements were made in water; otherwise a rather large correction would be required.

(c) The data of V. C. Wilson[16] were normalized in the neighborhood of 100 m.w.e. to the smooth curve published by Rossi.[17] This curve, however, made use of the same set of data by Wilson and was normalized at sea level to an absolute determination of the hard component intensity, 0.83×10^{-2} cm^{-2} sec^{-1} sterad^{-1}. No further correction was made, however, for the fact that Wilson used no absorber between the counters, and that the amount of soft component in equilibrium with the penetrating radiation changes with depth.

(d) The data given by Clay and Van Gemert[1] were taken with several different counter telescopes, but we chose to use only the data from one of their sets, a threefold coincidence set with layers of lead 5 cm thick between the counter trays, which were spaced close together and shielded by 5 cm Pb on all sides. At all but two of the depths, the authors reported measurements both with and without the two 5 cm Pb layers between the counters. At the remaining two locations, only the rates without this absorber were measured. These we corrected to rates with absorber by comparison with the data at other depths. Thus the points ascribed to Clay and van Gemert should be comparatively free of error due to secondary electrons. These points were normalized at 100 m.w.e., like the data of Wilson, to the graph published by Rossi.

(e) The reported data of Miyazaki[8] were difficult to understand, particularly since inconsistencies in the report indicate the presence of typographical or other errors. Also it seemed to us that his apparatus was sensitive to local secondaries; and his assumed zenith angle distribution, used in calculating the absolute intensities, was wrong. However, his data extend to a greater depth than any other data, and it seemed clear that the intensity of penetrating particles at 3000 m.w.e. was about 12 times less than at 1400 m.w.e. Therefore we normalized his data at 1400 m.w.e. to the smooth curve determined by the other data in Fig. 5, and used his ratio of intensities at 3000 and 1400 m.w.e. to plot the point at 3000 m.w.e.

(f) The intensities reported by Randall and Hazen[3] and by Bollinger,[7] as well as the intensity at 1574 m.w.e. reported here, are all determined by absolute measurements with lead filters to absorb the soft component. These points were not renormalized in any way. The absolute measurements are indicated on the graph by double circles.

The slight discrepancies that exist among the data plotted in Fig. 5 can be accounted for by differences in the composition of the ground in different locations, differences in the apertures of the telescopes and variation of the zenith angle distribution with depth, errors in the normalization, and effects associated with the soft component present underground. The best fit to the data in Fig. 5 seemed to be given by a curve with a continuously increasing slope, and accordingly such a curve was drawn. There was no sound evidence for a "knee" or abrupt change in slope; sharp bends can in any case be ruled out by physical arguments.

The slope of the smooth curve in Fig. 5 is plotted as a function of depth in Fig. 6. The accuracy with which the slope is known depends on the assumption of smoothness. An error of 0.1 in m extending over a large fraction of the curve in Fig. 6 would correspond to a noticeable discrepancy with the data in Fig. 5, but a considerably larger error could be tolerated over a short range of depth. For this reason, the value of m is particularly in doubt beyond 2000 m.w.e., but we estimate its uncertainty as 0.1 to 0.2 at smaller depths. The approximate region of uncertainty is indicated by the shaded area in Fig. 6. The value of m given by Bollinger,

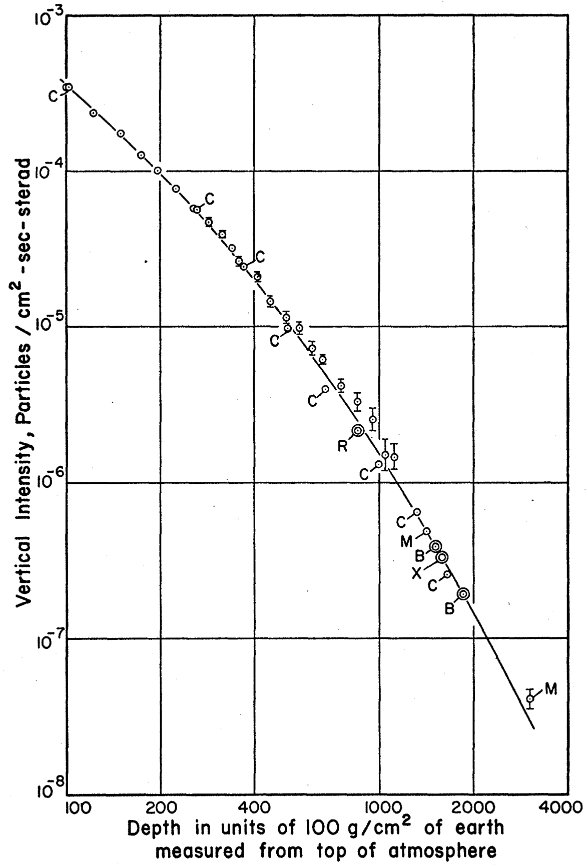

FIG. 5. Vertical intensity as a function of depth. Points not labeled are derived from V. C. Wilson, reference 16. Points designated C are from Clay and van Gemert, reference 1; M, Miyazaki, reference 8; R, Randall and Hazen, reference 3; B, Bollinger, reference 7; and X, the present experiment. The points enclosed in double circles are absolute measurements, not normalized.

[16] V. C. Wilson, Phys. Rev. **53**, 337 (1938).
[17] B. Rossi, Revs. Modern Phys. **20**, 537 (1948).

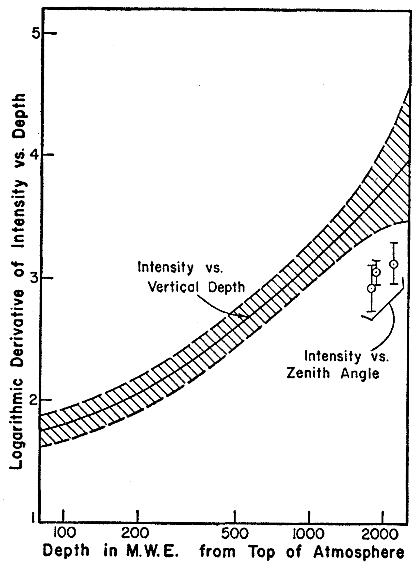

FIG. 6. The solid curve represents as a function of vertical depth the slope of the smooth curve drawn in Fig. 5; i.e., the quantity $m=-(h/I)(dI/dh)$. The shaded region indicates the estimated uncertainty in m. The circles represent the derivative $n=(\cos\theta/I)(dI/d\cos\theta)$, as given by reference 7 and the present experiment. The difference $m-n$ shows the effect of the decay process or processes in which the μ-mesons are created.

3.5 ± 0.3 at a depth of 1670 m.w.e., is in good agreement with the curve.

To compare with these values of m, three values of n, two given by Bollinger[7] and one by the present experiment, are plotted as circles in Fig. 6. The mean value of $\cos\theta$ used in the determination of n was in each case about 0.85; therefore each point is plotted at a depth equal to $h/0.85$, h being the vertical depth at which the measurement was made.

It seems that a real difference exists between n and m in the expected direction, indicating competition between decay and absorption of the parents of the particles detected underground. However, because of the uncertainty in m, the difference $m-n$ is not accurately determined. Its value is 0.6 ± 0.3, which could be consistent, according to Fig. 4, either with production of the μ-mesons only by $\pi-\mu$ decay, with production by equal numbers of π- and κ-mesons, or even with production solely by $\kappa-\mu$ decay, provided the estimated parameters of this process are correct. The agreement is best with the results expected ($m-n=0.5$) if equal numbers of π- and κ-mesons are produced with both types contributing to the μ-meson intensity. In the future, improved knowledge of the lifetime and decay scheme of κ-mesons, and of the intensity as a function of depth underground, might permit distinguishing between the alternatives, provided still more alternatives do not arise among which one must choose.

For the present, one can only rule out direct production of the μ-mesons or production through a decay of a meson with a lifetime short compared with 10^{-9} sec. If such processes occurred with frequency comparable to π-meson production, they would become the dominant processes in creation of μ-mesons that penetrate 1600 m.w.e., because of the small probability ($\sim$10 percent) of $\pi-\mu$ decay at energies around 10^{12} ev. The expected value of $m-n$ at 1600 m.w.e. would then be significantly less than the difference observed. As shown below in Section E, the interactions to which these conclusions apply are those caused by primaries of $10^{13}-10^{14}$ ev, in which the secondary mesons have energies around 10^{12} ev.

Two other measurements of the zenith angle distribution, at 730 and 850 m.w.e., were discussed above, with the conclusion that at about 800 m.w.e. the correct exponent n lay between 1.7 and 2.8. Taking the effective depth as 800/0.85, one finds in Fig. 6 that the corresponding value of m is 3.1 ± 0.15. Again it seems there is a difference $m-n$ in the direction indicating production of the particles by a decay process, but the magnitude of the difference is too uncertain to allow further conclusions.

C. COMPARISON OF TWOFOLD AND FOURFOLD COINCIDENCE RATES

The evidence against the belief that the penetrating particles underground are μ-mesons has consisted chiefly of two expermental findings:[4] a strong absorption in lead within a counter telescope, with maxima and minima in the absorption curve; and a large ratio between twofold and threefold coincidences in a counter telescope, which suggested that the particles have low efficiency in producing ionization within a counter. The absorption curve has been discussed in Section A, and conclusive evidence has been presented, contradicting the occurrence of strong absorption or maxima and minima in the curve. In this section evidence is presented that the large ratio of twofold to threefold coincidences was also an illusion, and that the ratio is really almost exactly one when enough shielding is used to prevent twofold coincidences due to local radioactivity.

Evidence of this sort has already been presented by Miesowicz *et al.*[5], Tiffany and Hazen,[2] and Bollinger.[7] Our present results will therefore only confirm previous experiments contradicting the low ionization efficiency. Furthermore, the data given in Section A showed that the penetrating particles which produce fourfold coincidences have at least 99 percent efficiency in discharging a fifth counter. This failed to be a conclusive contradiction only because of the possibility of the existence of a second penetrating component which, because of low ionization efficiency, is seldom recorded by fourfold coincidences.

In order to study the parallel penetrating particles occurring underground and the time variations of the intensity, five counter arrangements were set up like the one shown in Fig. 7. Each of these sets had a sensitive area of about 30 in.$\times$40 in. The upper tray was

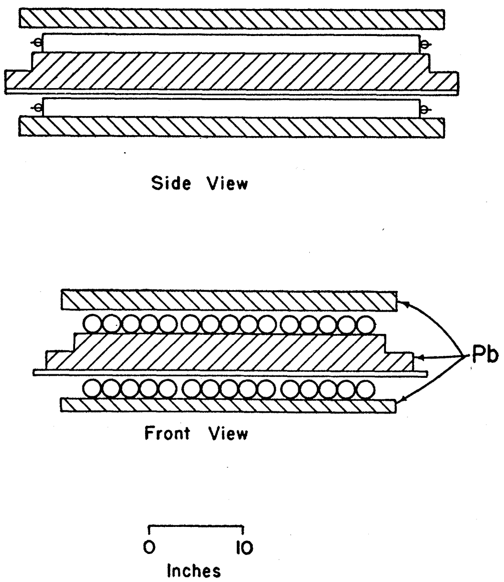

FIG. 7. One of the five telescopes used in the study of meson showers at 1600 m.w.e.

separated from the lower one by 4-in. Pb+0.5-in. Fe, and 2-in. Pb absorbers were also placed above the upper tray and below the lower tray to reduce the effects of soft component. In each of the five sets of apparatus, the twofold coincidence rate between the upper tray and the lower tray was recorded by a mechanical register. (Hodoscope photographs, showing which counters were struck, were only taken when there was a coincidence between two of the sets or when at least two counters were discharged in both the upper and the lower tray of one set.)

The average coincidence rate was 10.46±0.04 per hour. Of this rate, the number of accidentals was about 0.5±0.3 per hour, the large error being due to uncertainty in the single counting rates and the effective resolving time for chance coincidences. Thus the corrected rate is 10.0±0.3 per hour, due to cosmic rays penetrating 4-in. Pb+0.5-in. Fe and producing twofold coincidences. (The uncertainty of 0.3 in this rate should be regarded as an upper limit of error.)

The absolute intensity, at the same depth, of cosmic rays traversing three absorbers totalling 12-in. Pb+1.5-in. Fe, and producing fourfold coincidences, was found in Section A to be $(3.31\pm0.04)\times10^{-7}\times(\cos\theta)^{3.00\pm0.09}$.§ With this expression, and with the assumptions that only these particles produce the twofold coincidences in the other counter sets, and that the ionization efficiency is 100 percent, a calculation was made of the expected twofold coincidence rate. The result was 10.2±0.2 per hour, in good agreement with the rate observed. Therefore the number of penetrating particles producing twofold coincidences with low ionization efficiency at 1574 m.w.e. is less than about one percent of the number of ionizing penetrating particles.

These data also confirm that the absorption between 4-in. and 12-in. Pb is less than about one percent, consistent with the results of Section A. The expected absorption in 8-in. Pb+1-in. Fe is 0.4 percent on the assumption that the particles are μ-mesons and are not local secondaries but comprise the penetrating component itself.

D. VARIATION OF THE INTENSITY WITH ATMOSPHERIC TEMPERATURE

The μ-mesons observed at sea level exhibit a negative correlation with atmospheric temperature, as a consequence of their decay in the atmosphere. At energies sufficient for penetration to 1600 m.w.e. this effect is completely negligible; but if the μ-mesons arise from a decay process, there should be a positive temperature effect owing to the competition between decay and nuclear absorption of the parents. A rise in temperature implies an atmospheric expansion, which increases the probability that a high energy parent meson will decay before being absorbed, and thus increases the production of μ-mesons. It is assumed that only μ-mesons can penetrate to great depths underground.

The magnitude of the temperature effect at a given depth should depend on the mean lifetime for the decay process. Therefore measurement of this effect can in principle provide another test of the identity and mode of origin of the particles that penetrate far underground. In this regard, measurement of the temperature effect is similar to measuring the difference between the exponents in the zenith-angle dependence and the vertical depth dependence of the underground intensity.

Equations describing the expected temperature effect are derived in the Appendix, and the results are shown in Fig. 8. If the atmosphere were isothermal with absolute temperature T, the temperature coefficient α

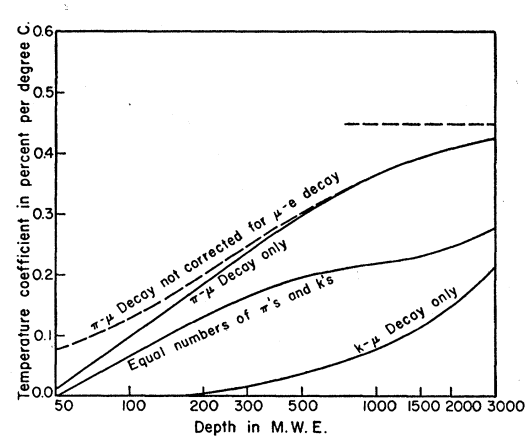

FIG. 8. Calculated temperature effect as a function of depth, under various hypotheses as to the nature of the parents of the μ-mesons. The solid curves take into account the effect of $\mu-e$ decay, while the dashed curve illustrates the magnitude of this correction. The calculations are explained in the Appendix.

§ We use the values obtained before the small corrections for secondaries because this error should be similar in both experiments.

would be defined by $\alpha=(1/I)(\partial I/\partial T)$, where I represents the intensity.

The atmosphere, however, is not isothermal. It is shown in the Appendix that the temperature distribution can be taken into account by using a weighted average temperature in the above formula. In this average, pressure levels near the top of the atmosphere must be weighted heavily in comparison with the pressure levels at lower altitude, for two reasons: (1) a high energy parent meson has very little chance to decay after it reaches the high density regions at low altitude, and (2) very few of the parent mesons survive both nuclear absorption and decay long enough to reach low altitude. A sufficiently close approximation to the correct weighting factor is $\exp(-x/\lambda_p)dx$, where x is the atmospheric pressure and λ_p is the absorption mean free path of the primary cosmic rays.

Temperature information was supplied by the U. S. Weather Bureau from radiosonde observations made by the Air Force station at Rome, New York, 75 miles northeast of the salt mine in which our apparatus was located. For comparison, data were also obtained from the Weather Bureau stations at Buffalo and Albany, New York, respectively about 120 miles WNW and 145 miles east of the salt mine. The variations of temperature were found to be very similar at the three stations, and the Rome data showed a tendency to lie between the data from Albany and Buffalo. Therefore we believe no significant error arises from our assumption that the temperature fluctuations measured over Rome are the same as those over Ithaca.

The flights were made four times each day, beginning at local times 0400, 1000, 1600, and 2200 hours. However, the flights made at night consistently failed to reach as high altitudes as those attained during the daytime. Hence our information on the diurnal temperature variations does not extend above 100 mb. Fortunately the average diurnal temperature variation decreases

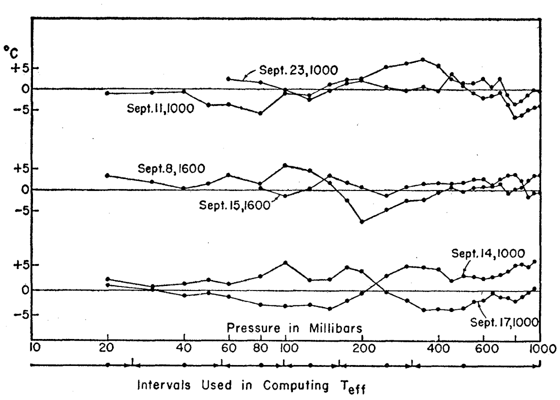

FIG. 9. Illustrations of the non-uniformity of temperature fluctuations in the atmosphere. For six representative balloon flights, the figure shows, as a function of pressure, the deviation of the temperature from the average value at the same pressure level. The scale at the bottom shows the pressure intervals into which the atmosphere was divided in computing "effective" temperatures.

rapidly with increasing elevation, being large only near sea level and essentially zero in the upper half of the atmosphere. At least one daytime flight reached the 20-mb level on about half of the days.

The weighted average or effective temperatures of the atmosphere, T_{eff}, were computed by the following approximate method. The atmosphere was divided into a set of unequal pressure intervals Δx such that $\Delta \exp(-x/\lambda)$ was constant, with $\lambda=120$ g/cm^2. Within each interval a fixed point was chosen at which temperature data were available. T_{eff} was then computed as the linear average of the temperatures at these points. The interval at lowest altitude was only half a normal interval by our criterion; hence the temperature in this interval was given a weighting factor $\frac{1}{2}$. The pressure intervals selected, and the points chosen to represent the intervals, are shown on the scale in Fig. 9. The formula used for T_{eff} was

$$T_{\text{eff}}=(T_{20}+T_{40}+T_{80}+T_{125}+T_{250}+\tfrac{1}{2}T_{500})/5.5.$$

A day-to-day correlation of intensities with T_{eff} or with the temperature at any pressure level of the atmosphere was rejected as unreliable for the following reasons: (1) Temperature information at two of the important levels, 20 and 40 mb, was obtained at most twice per day and not at all on about half of the days. Even at 80 mb, data were seldom obtained at night. (2) Large changes in temperature frequently occurred within 6 hours at high levels of the atmosphere. These fluctuations sometimes were almost as large as the greatest temperature changes recorded in the course of a year. Therefore the temperature could not be assumed known during the intervals between successive measurements.

These difficulties were overcome by basing the correlation on monthly averages of the temperatures and intensities. Except for a seasonal trend, the temperature fluctuations at high levels seemed to be random, and since the temperature at each level was sampled many times in the course of a month, the average temperature could be considered well known.

The use of monthly averages also reduces two other sources of small errors. From day to day the temperatures at Rome, New York, probably fluctuated slightly differently than at Ithaca, but in the monthly averages some of these differences would be canceled. Secondly, the approximate calculation of T_{eff}, using the temperatures at a small number of points, causes random errors from day to day which cancel in a long-time average.

In Fig. 9 a number of typical sets of temperature data from individual flights have been plotted. The ordinate of each point in this figure is the deviation of the temperature reading on that particular flight from the average temperature for the month at the same pressure level. Thus the normal change of temperature with elevation does not appear in the figure. Such graphs often show that day-to-day temperature deviations at a single level are compensated for by deviations in the opposite direction at other levels. Since the mesons are produced throughout the atmosphere instead of at a single level, one must therefore expect poor correlation between the intensity and day-to-day fluctuations of temperature at a single level.

In Fig. 10 the monthly average temperatures at various levels of the atmosphere have been plotted, for the time within which intensity data were collected. The lowest graph shows the seasonal variation of T_{eff}. During the time of our measurements, the temperatures in the range 80–125 mb exhibited properties different

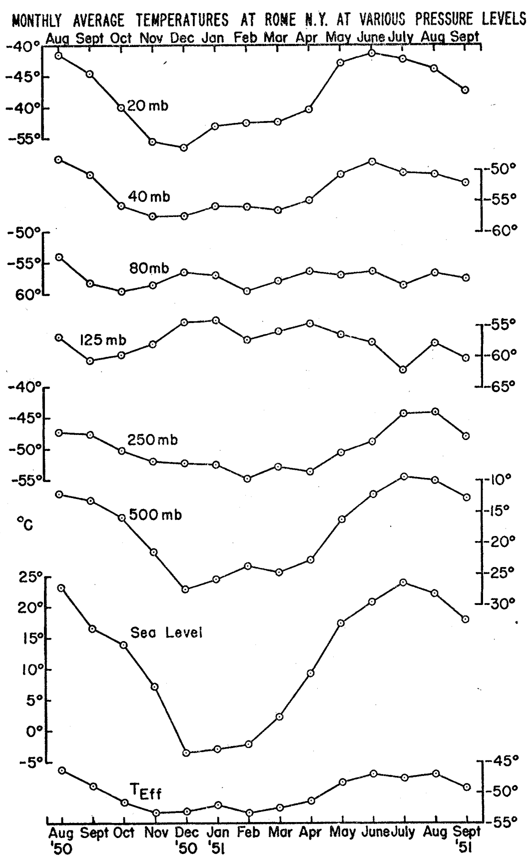

FIG. 10. Seasonal variation, during 1950–1951, of the "effective temperature" of the atmosphere and the temperature at various pressure levels.

from those of the other levels. Firstly, they were the lowest temperatures, even lower than those at 20 and 40 mb. Secondly, their seasonal variation was out of phase with the seasonal variation above and below this pressure range. For instance the correlation coefficient r between the monthly average temperature of the 100-mb level and the monthly average of T_{eff}, during the time from August, 1950 to October, 1951, over Rome, was

$$r(T_{100}, T_{\text{eff}}) = 0.061,$$

which is practically zero correlation, in spite of the fact that T_{80} and T_{125} were used in computing T_{eff}.

There were two series of intensity measurements, which we shall call Series I and Series II. Series I ran during the months August, 1950 to February, 1951 inclusive, during which 20,445 counts were recorded with the apparatus shown in Fig. 1. Each count was registered on clock-driven paper in an Esterline-Angus recorder. A check of the counting was provided by a mechanical register, and another check by the hodoscope pictures taken of all the events. The Geiger counters behaved with perfect stability, and chance coincidences were completely negligible.

Series II ran from July 15 to October 10, 1951, in which time 90,702 counts were recorded by five telescopes like the one shown in Fig. 7. The coincidences in each telescope were added on a mechanical register, and the set of five registers was photographed every hour. About five percent of the counts were due to chance coincidences, since only twofold coincidence was required for each count; but this background has so little effect on the variation of rate with temperature that no correction was applied.

The data from both series of measurements are given in Table III. The five telescopes used in Series II differed appreciably in mean counting rate, because the counters were not quite identical in size, spacing, and background rate; the operating times also were not all the same; therefore the counts recorded in the five telescopes are listed separately.

For Series I and each telescope of Series II (i.e., for

TABLE III. Counting rates *vs* effective temperature of the atmosphere.

Month	ΔT_{eff} (°C)	Series	t (hours)	n (counts)	$\Delta n = n - \bar{R}t$
1950 Aug.	+4.9	I	491	2500	+ 82
Sept.	+2.2		604	2954	− 21
Oct.	−0.3		578	2923	+ 76
Nov.	−2.0		670	3216	− 84
Dec.	−1.8		702	3446	− 12
1951 Jan.	−0.8		707	3411	− 71
Feb.	−2.0		399	1995	+ 30
1951 July 15–31	+1.1	II (a)	358	3789	+ 77
		(b)	165	1824	+ 47
		(c)	302	3203	+ 93
		(d)	358	3550	+ 55
		(e)	358	4044	+ 37
1951 Aug.	+1.2	II (a)	618	6352	− 56
		(b)	294	3291	+124
		(c)	618	6308	− 57
		(d)	618	6059	+ 27
		(e)	618	7083	+167
1951 Sept.	−0.9	II (a)	655	6792	0
		(b)	691	7412	− 31
		(c)	691	7109	− 8
		(d)	691	6738	− 7
		(e)	688	7525	−175
1951 Oct. 1–9	−2.3	II (a)	167	1710	− 22
		(b)	195	1961	−140
		(c)	195	1980	− 28
		(d)	194	1819	− 75
		(e)	195	2153	− 29

Series I: $\bar{T}_{\text{eff}} = -51.2°$, $\bar{R} = 4.93$ hr^{-1}.
Series II: $\bar{T}_{\text{eff}} = -48.3°$, $\bar{R} = 10.37$ hr^{-1} (a), 10.77 (b), 10.30 (c), 9.76 (d), and 11.19 (e).

$j=1$ to $j=6$) one can obtain an average rate

$$\bar{R}_j=\sum_i n_{ij}/\sum_i t_{ij}$$

and an average temperature $\bar{T}_j=\sum_i t_{ij}T_{ij}/\sum_i t_{ij}$. In the individual time intervals t_{ij}, the numbers of counts n_{ij} differ from the expected numbers $\bar{R}_j t_{ij}$ by Δn_{ij}, and the temperatures differ from $\bar{T}_j$ by ΔT_{ij}. We assume that the temperatures T_{ij} are accurate; any error in this assumption is only likely to reduce the calculated temperature effect. Further, we assume that there are no causes of variation of the rate R_{ij} other than temperature changes and random statistical errors. This assumption can be checked after computation of the temperature effect, by seeing whether the residual fluctuations are as small as expected from statistical errors only.

Under these assumptions, the most probable value of the temperature coefficient α and its standard error are given by

$$\alpha_0=\frac{\sum_j\sum_i\Delta n_{ij}\Delta T_{ij}}{\sum_j\sum_i\bar{R}_j t_{ij}(\Delta T_{ij})^2}\pm\frac{1}{[\sum_j\sum_i\bar{R}_j t_{ij}(\Delta T_{ij})^2]^{\frac{1}{2}}}.$$

The corresponding equation for the correlation coefficient r between counting rates and temperature is

$$r=\frac{\sum_j\sum_i\Delta n_{ij}\Delta T_{ij}}{[\sum_j\sum_i(\Delta n_{ij})^2/\bar{R}_j t_{ij}\cdot\sum_j\sum_i\bar{R}_j t_{ij}(\Delta T_{ij})^2]^{\frac{1}{2}}}.$$

Let Δn^*_{ij} be the residual deviation after correction of the rates with a temperature coefficient α': i.e., $\Delta n^*_{ij}=\Delta n_{ij}-\alpha'\bar{R}_j t_{ij}$. The sum of the squares of the reduced deviations Δn^*_{ij}, relative to the standard statistical errors $(\bar{R}_j t_{ij})^{\frac{1}{2}}$, is related to the sum using uncorrected deviations, and to the temperature coefficient α', by

$$S(\alpha')=\sum_j\sum_i\frac{(\Delta n^*_{ij})^2}{\bar{R}_j t_{ij}}=(1-r^2)\sum_j\sum_i\frac{(\Delta n_{ij})^2}{\bar{R}_j t_{ij}}+\left(\frac{\alpha'-\alpha_0}{\epsilon_0}\right)^2,$$

where r is the correlation coefficient, α_0 is the most probable value of the temperature coefficient, and ϵ_0 is the standard error in α_0, given above. For $\alpha'=\alpha_0$, the expected value of $S(\alpha')$, if the assumptions made above are correct, is

$$\langle S(\alpha_0)\rangle=m-m_j-1\pm[2(m-m_j-1)]^{\frac{1}{2}}$$

where m is the total number of terms in the double sum and m_j is the number of series of measurements or number of terms in the sum over j. (The subtraction of m_j+1 from m is due to the fact that the choice of the $\bar{R}_j$ and of α_0 minimizes the sum S to a value below that which would be expected if the "true" values of $\bar{R}_j$ and α were used.)

The above formulas can be applied separately to the data of Series I and to those of each of the five counter sets of Series II. When this is done, all six sets of data yield positive correlation coefficients and positive regression coefficients, which are in reasonable agreement, considering the standard errors; but the errors are large and the individual results are not highly significant. Alternatively, the relations can be applied to the complete set of data, by which means the following results are obtained: $\alpha_0=0.79\pm0.20$ percent per degree C, $r=0.60$, $S(\alpha_0)=27.6$, and $\langle S(\alpha_0)\rangle=20\pm6.3$ (expected value).

Thus, the reduced deviations are equal to those expected from purely statistical errors, within the expected statistical fluctuation of the deviations. This fact gives support to the assumptions on which the equations for α_0 and its error were based. The most probable value of α is almost four times its standard error, and therefore the positive temperature effect is considered to be real.

A certain reservation must be applied in considering the error in α_0. It is possible to conceive of systematic changes in intensity ("secular" variations) which by chance have such periods and phase as to cause part of the apparent temperature effect, without influencing strongly the sum of reduced deviations $S(\alpha_0)$. That is, the secular variations might by chance be correlated with the changes in temperature. If one does not know the causes of the secular variations one has no means of estimating their possible effect, except qualitatively that they must increase the uncertainty in α.

One type of variation which could do this is a change in the efficiency of the apparatus. In our case, however, there were six independent counter sets and recording systems, all of which yielded similar results.

Other variations would have to lie in the intensity of the cosmic rays themselves. In our experiments the energies of the particles are so high, that known causes of intensity variations, such as changes in barometric pressure, the earth's magnetic field, and solar magnetic activity would be negligible.

Other, unknown effects may have influenced the data, but it is unlikely that these influences would repeatedly be in phase with the temperature changes. The data given above were taken in two successive years. In order to test the results further, a new apparatus has been constructed which will be operated for another year and which will have better statistical accuracy.

N. Sherman and W. E. Hazen have kindly informed us of preliminary results of a similar experiment performed at a depth of 850 m.w.e. in a salt mine in Michigan. With a statistical accuracy even better than ours, they find α_0 to be practically zero. This result cannot be reconciled with ours, nor with any known mechanism of origin of μ-mesons (unless the production of π-mesons and of heavy mesons which decay into π's practically ceases at 10^{11} ev).‖

‖ The hypothesis that direct secondaries of nuclear interactions are themselves the particles which penetrate far underground (see reference 50) is ruled out by recent experiments on the interaction of high energy shower particles in emulsions. See, for instance, Lal, Pal, Peters, and Swami, Proc. Indian Acad. Sci. (to be published).

On the other hand, our value of α_0 is larger than the maximum possible value, as may be seen by comparing with Fig. 8. However, if all or most of the μ-mesons originate by $\pi-\mu$ decay, the expected value of α_0 differs from our experimental value by only two standard errors, which is not a very improbable fluctuation.

In Section B, a difference was found between the dependence of intensity on vertical depth and the dependence on cosine of the zenith angle. That result and the present measurement of the temperature effect support each other in indicating the influence of a decay process in the origin of the particles which penetrate far underground. Both experiments are reasonably consistent with the hypothesis that all of these particles are produced by $\pi-\mu$ decay, but do not exclude the possibility that some of the particles arise by $\kappa-\mu$ decay.

Diurnal Variations

Since the lower part of the atmosphere exhibits a large diurnal temperature variation while the upper part exhibits very little, evidence of a diurnal intensity variation with the right phase would indicate significant production in the lower atmosphere of the particles capable of penetrating far underground. However, neither the rates in Series I nor those in Series II exhibited any significant variation with solar time. This result has been discussed elsewhere in relation to the origin of cosmic rays.[18,19] In order to obtain the statistical limits of a possible intensity variation in phase with the sea level temperature (maximum at 1440 hours), we have attempted to fit the counting rates R_i recorded in Series II at different times of the day, t_i, to the equation

$$R_i = A + B\cos[2\pi(t_i - 1440)/2400].$$

The method of least squares yields $B/A = 0.0028$ with a standard error of 0.0047. During Series II, the mean amplitude of the diurnal temperature variation at sea level was 5.2°. Therefore the sea level temperature coefficient indicated by our data is

$$\alpha_{SL} = 0.05 \pm 0.09 \text{ percent per degree C},$$

the small value of which supports our belief that significant production of high energy mesons occurs only in the upper part of the atmosphere.

Previous Measurements of Temperature Effect

The earliest report of a positive temperature coefficient, to our knowledge, was that of Forró,[20] who found that the intensity at 1000 m.w.e. depended on sea level temperature with a coefficient $\alpha_{SL} = 0.74 \pm 0.19$ percent per degree C (after converting her probable error to a standard error for consistency with the present report). As already mentioned by Forró, this result is unreasonable in view of the relative changes of temperature at high altitudes as compared with those at sea level, and the maximum possible magnitude of the temperature effect, which is the reciprocal of the absolute temperature. Besides, the result was based on only 2056 counts. A recalculation by our method, using Forró's data, leads to different values of the coefficient and its standard error; namely, $\alpha_{SL} = 0.44 \pm 0.31$ percent per degree. This is in better agreement with the expected value of an α_{SL} based on seasonal temperature changes, which is about 0.1 percent per degree, taking $\alpha_{\text{eff}} \simeq 0.4$ percent per degree and the seasonal temperature change at sea level as about five times the change in T_{eff}.¶

Attempts have also been made to gather evidence about the mechanism of production of μ-mesons of much lower energy, $10^9 - 10^{10}$ ev, by measuring the temperature effect in the intensity of penetrating particles at sea level. In such experiments, the methods applied are such as to cancel out the effect of $\mu-e$ decay in the atmosphere, which decay depends on the height of the production layers. Duperier[21] has found an indication of a positive correlation with the mean temperature of pressure intervals near the top of the atmosphere. His analysis of the significance of this result seems to be in error because of the approximations that production occurs at a single level, and that the density of the atmosphere is constant over the distance in which $\pi-\mu$ decay competes with nuclear absorption. Cotton and Curtis[22] have found no correlation between the sea level intensity and the temperature of the 100-millibar level, contradicting the results of Duperier. Any search for a positive temperature coefficient in the sea level intensity suffers from the following difficulty: it is not the temperature of a single layer or narrow band of the atmosphere that matters, but a weighted average of temperatures throughout the whole atmosphere. This is so closely related to the heights of the pressure levels that the height correlation associated with $\mu-e$ decay and the temperature effect associated with production of the μ-mesons cannot be entirely separated. For the intensity far underground,

[18] G. Cocconi, Phys. Rev. **83**, 1193 (1951).
[19] P. H. Barrett and Y. Eisenberg, Phys. Rev. **85**, 674 (1952).
[20] M. Forró, Phys. Rev. **72**, 868 (1947).

¶ Apparently, Forró used a statistical treatment similar to that outlined by Janossy and Rochester in Proc. Roy. Soc. (London) **A183**, 186 (1944). This method breaks down seriously when the individual series of measurements contain very few terms (as, for instance, three terms per series in the data of Forró). One reason is that a term in the calculation of the error, given as $(n-1)$, should be $(n-2)$, a significant change when n is only 3. A more important reason is that in each series the error was inferred from the consistency with which the three points happened to fit a straight line, rather than from *a priori* knowledge of the statistical errors. In some of the series, the fit happened by chance to be very good, as must occasionally be expected. In these cases the computed error was much too small. In a weighted average of all the series, therefore, these cases may be given undue influence. Thus the final result can be heavily biased by a small fraction of the data, and can also have a computed error considerably less than that which must be due to randomness alone.

[21] A. Duperier, Nature **167**, 312 (1951).
[22] E. S. Cotten and H. O. Curtis, Phys. Rev. **84**, 840 (1951).

the situation is much simpler, because of the improbability of $\mu - e$ decay, and the insignificance of absorption of the μ-mesons in the atmosphere.

E. ASSOCIATION OF THE PENETRATING PARTICLES WITH EXTENSIVE AIR SHOWERS

μ-mesons of moderate energy in the atmosphere are sometimes found to be associated with extensive air showers containing many photons and electrons. However, the fraction of the mesons having a detectable coherence with air showers is very small, about 0.6×10^{-3} according to the estimate published by Cocconi.[23] The production of mesons is frequently multiple in single nuclear interactions, and the multiplicity is enhanced by further nuclear interactions of the secondary mesons and nucleons. Some of the mesons produced are π° mesons which decay into photons, which in turn generate the electronic shower. Many of the charged mesons decay and thus give rise to the muons. This process may possibly be the unique process of μ-meson production. However, the majority of the primary cosmic rays do not have enough energy to generate a very large shower. Hence the electrons and mesons are usually few in number. The electrons and the N component are rapidly absorbed in the atmosphere, and the few particles that remain are spread by scattering over an area of thousands of square meters; thus they are usually observed as single particles. The μ-mesons, having greater ranges than the N component and the electronic component, are the particles most likely to survive the rest of the shower and to be spread far apart from each other. Since coherence between particles is seldom observed when the particle density is less than about one per square meter, the mesons are almost always observed as single particles (except for an accompaniment by locally generated knock-on electrons).

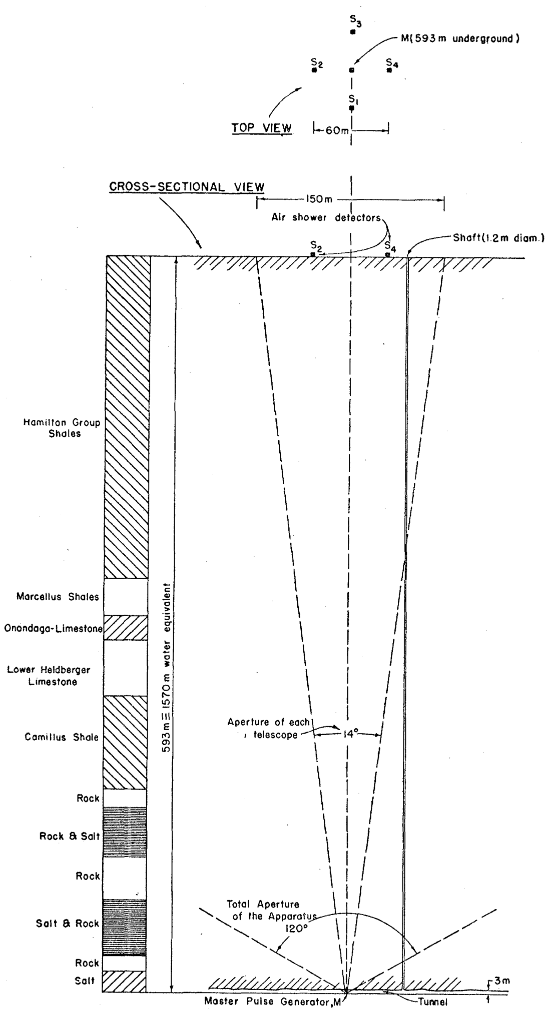

FIG. 11. Diagram showing relative positions of the counters underground and on the surface of the ground in the experiments on the association of mesons with extensive air showers. The composition of the ground is shown in the scale at the left.

[23] G. Cocconi, Phys. Rev. **79**, 1006 (1950).

If the particles detected in the salt mine are μ-mesons created in the air, which require about 10^{12} ev to penetrate to 1600 m.w.e., the association with extensive air showers should be easier to detect for two reasons: (1) the primary energy would be large enough to generate a shower which still contains many electrons at sea level, and (2) the mesons would be very little scattered, hence would be detected close to the core where the electron density is high.

Therefore apparatus was arranged as shown in Fig. 11, which also contains some information about the composition of the ground. In the passage of the mine, 594 meters underground, was located the counter array shown in Fig. 1 and discussed in Section A. Fourfold coincidences of trays A, B, C, and D were made to generate master pulses, which were sent to the surface, along with information as to which counters had been discharged underground, by means of cables in the shaft shown in Fig. 11. The four counter boxes on the surface of the ground each contained 15 counters of 492 cm^2 area and 3 to 5 counters of 30 cm^2 area, making a total sensitive area of 3 m^2 distributed at points on a horizontal circle of 30-m radius. The master pulse was put in coincidence with each of the counters, and coincident discharges were indicated by the lighting of neon bulbs in the same hodoscope that also showed which counters had been struck underground.

Switches located with the hodoscope at the "central station" made it possible to check the operation of each counter regularly, both at normal operating voltage and at a reduced voltage, without going underground or opening the counter boxes above ground. The latter boxes were constructed of thin insulating material and were provided with temperature-regulated heating. The switches at the central station also permitted checking the continuity of all the individual lines, and provided tests of the operation of the coincidence and pulse forming circuits. After an initial period of trouble-

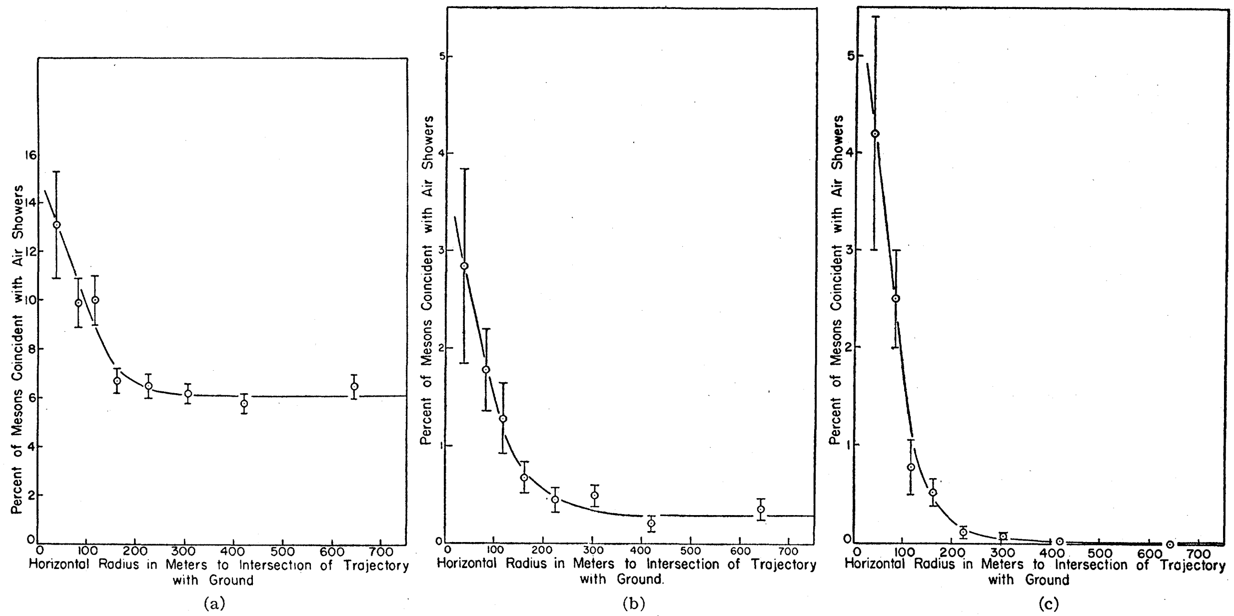

FIG. 12. Association of mesons at 1600 m.w.e. with extensive air showers on the surface of the ground: (a) showers that discharge single counters, (b) showers that discharge two counters, (c) showers that discharge more than two counters. The events with radii greater than 400 meters are due to chance coincidences. Curves are empirical.

shooting, the whole apparatus functioned smoothly for a period of six months, during which time 18,098 good hodoscope pictures were taken.

Dependence of Coincidences on Direction of Meson

Only a small fraction of the pictures showed coincidences between the master pulses and the unshielded counters on the surface of the ground. The principal reason for this becomes obvious upon glancing at Fig. 11. The counters above and below ground amount to a telescope of 600-meter length. Most of the mesons that were detected underground travelled at zenith angles greater than 20° and hence missed the counters on the surface by more than 200 meters. Air showers with cores striking this far from the counters would have to be of very great size to be detected with appreciable efficiency.

The hodoscope records, however, provided a means of sorting the mesons according to zenith angle. In Figs. 12 and 13 is plotted the percentage of mesons associated with detected air showers, as a function of the horizontal distance between a point directly over the meson detector and the point where the meson's path seemed to intersect the ground. As might be expected, when this distance was greater than about 400 meters no showers were detected, the only coincidences being due to chance (the rate at large distances agrees well with the chance background measured independently and also computed from the circuit resolving time); but as the distance decreased toward zero, the probability of detecting the air showers increased rapidly.

Indeed, the imperfect angular resolution of the counter telescopes underground (± 7 degrees in two directions) tends to smooth out the angular dependence of the rate of coincidences between mesons and air showers. Also, because of the separation of the counter boxes, no shower core could strike at an average distance from the counters less than 30 meters. Therefore Figs. 12 and 13 present a distorted distribution, and indicate that an unmodified distribution would be steeper and would rise higher as the distance approaches zero.

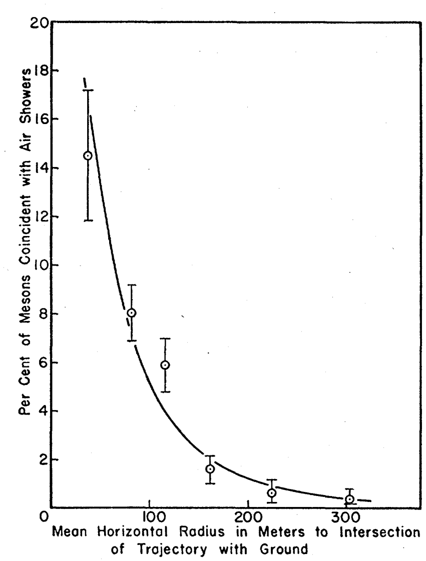

FIG. 13. Association of mesons at 1600 m.w.e. with extensive air showers striking one or more counters on the surface of the ground, after correction for chance coincidences. The curve has been calculated as described in the text.

The most vertical telescopes underground had a resolution function at the surface which was pyramidal in shape, centered relative to the unshielded counter boxes, and 150 meters square at the base, assuming no scattering of the mesons. Averaging over this function, the mean distance of meson paths from the counters was 44 meters. Of this group of particles, 14.5 ± 2.7 percent (after subtracting chance background) were *observed* to be associated with air showers. Many of the showers were so small that they struck only single counters. Therefore the probability of a shower escaping detection must have been high and we are forced to conclude that a much larger fraction, certainly at least 30 percent and probably all, of the mesons were indeed associated with air showers at sea level. At any rate, the *observed* association with air showers is 200 times greater than that of the lower energy mesons detected above the surface of the ground.[23]

The significance of the direction of the meson also appeared in the following observation. Usually when a meson was detected underground, some inclination was indicated, so it was possible to judge which of the four counter boxes on the surface should be closest to the path of the meson, provided the meson traveled in a perfectly straight line. For the showers striking more than 3 counters, of which none were expected by chance, this box was invariably the one in which the most counters were discharged. Thus the counter box closest to the path of the meson was also the one closest to the core of the air shower. This test could be applied to all but the quite vertical telescopes, hence it implies that the mesons were within 75 meters of the shower core at sea level, and were scattered through an angle less than 0.1 radian in the earth.

This degree of collimation implies high energy, not only of the mesons, but of the primaries of the nuclear interactions in which the parents of the μ-mesons are produced. If one assumes production at the 100-millibar level, 16 kilometers high, separations less than 75 m at sea level imply angles of less than 1/200 radian. From this, with an allowance for reasonable anisotropy in the center-of-mass system, we infer a Lorentz factor of at least 100 in the center-of-mass system of the nucleon-nucleon collision, and hence a primary energy of more than 2×10^{13} ev. Energies of this magnitude are also required to produce the observed air showers.

Of those mesons for which real associated air showers were not detected, a fraction (6.1 ± 0.2 percent) were by chance coincident with pulses in single shower counters; a smaller fraction (0.3 percent) were coincident with chance pulses in two shower counters; and so on, as evidenced by the backgrounds at large distances in Figs. 12(a)–(c). Similarly, 6 percent of the associated air showers striking single counters were coincident by chance with a particle hitting a second counter, and so forth. The corrections for chance coincidences therefore not only subtracted from the number of showers apparently associated with the mesons, but shifted the distribution of showers slightly towards smaller sizes. The corrected numbers of associated showers are listed in Table IV as a function of the number of counters discharged, and are plotted in Fig. 13 as a function of the distance of the meson trajectory from the shower counters.**

Average Number of Electrons in the Air Showers

The mean number of electrons per event can be calculated in a direct manner, without invoking assumptions likely to cause serious error. This possibility is due to the fact that the air showers are only detected when the associated mesons travel at small zenith angles θ, for which the approximation $\cos\theta=1$ is tolerable.

Let r be the perpendicular distance from one of the counter boxes on the surface of the ground to the trajectory of a meson detected at a vertical depth h; and assume that this trajectory also represents the path of the core of the associated air shower. The number of recorded mesons with trajectories lying between r and $r+dr$ will be

$$f(r)dr=I_vTA\cdot2\pi rdr/h^2=5.4\times10^{-6}\cdot2\pi rdr$$

where:

$I_v=3.3\times10^{-7}$ sec^{-1} cm^{-2} sterad^{-1}
=absolute vertical intensity of mesons at depth h,
$T=1.293\times10^7$ sec=running time of apparatus,
$A=4440$ cm^2=sensitive area of meson detector,
$h=5.94\times10^4$ cm.

Let $F(N)dN$ be the probability that there is an associated shower containing N to $N+dN$ electrons, and let $\varphi(N, r)$ be the density of electrons in a shower of N particles at a distance r from the core. Then the number of electrons, n_1, that strike the selected counter box in the course of the experiment is

$$n_1=\int_N\int_r f(r)dr\cdot F(N)dN\cdot S_1\varphi(N, r),$$

where we have let S_1 be the sensitive area of the counter box. To obtain the number of electrons recorded in all four boxes, n, one needs only multiply both sides of the equation by 4, or replace S_1 with the total sensitive area of all the unshielded counters, 2.97×10^4 cm^2. By

** Each point in Fig. 13 represents the sum of the frequencies of showers striking 1, 2, 3, $\cdots$ counters. The numbers of showers striking single counters has a large statistical error, because of the magnitude of the correction for chance coincidences; but the number has the property that it cannot be less than zero. The number of showers striking more than one counter required less correction and therefore is more accurately known. The sums plotted in Fig. 13 consequently have error probability distributions which are far from Gaussian, and are asymmetric. The true rates are more likely to be higher than the plotted error limits than lower. This difference is most pronounced for the two points at distances larger than 200 meters.

definition

$$\int_r \varphi(N,r)2\pi r dr = N$$

and

$$\int_N NF(N)dN = \bar{N}.$$

Therefore the equation for n reduces to

$$\bar{N} = 6.25n.$$

In Table IV we have listed the numbers of events, corrected for chance coincidences, in which various numbers of counters were discharged. The number n would be simply the sum of the counters discharged in all the events, except for the fact that sometimes more than one electron strikes a single counter. Each counter box contained 15 equal counters and several smaller counters that could be considered as a 16th equivalent counter. (Only when all 15 large counters were discharged was the resolution among the small counters necessary in calculating the density of electrons in the shower.) When q of the 16 counters were discharged, the approximate number k of electrons hitting the q counters could be calculated from the relation

$$q/(16-q) = (1-e^{-k/16})/e^{-k/16}.$$

By this means the numbers in the column 4 of Table IV were calculated. Their sum is 1082, with an rms error of about 240. This yields $\bar{N} = 6800 \pm 1500$ electrons per shower, under the assumption that every meson is associated with an air shower, not only the mesons for which such association was detected. The average number of electrons in the *detected* showers was larger than the $\bar{N}$ given above.

It should be emphasized that, because of the shape of the number distribution of the showers, the mean value of N is much larger than the median or typical value. In fact, if the distribution derived in the following paragraphs continued without growing steeper as $N \rightarrow \infty$, the mean value $\bar{N}$ would diverge. Hence we are required to use "average values" with caution.

The procedure outlined above was also applied to a selected number of the events listed in Table IV; namely, 37 events in which the counters underground seemed to indicate 2 or 3 parallel mesons simultaneously crossing the hodoscoped counter trays. Lack of obvious interactions of the particles in crossing about a meter of lead, the separation and parallel directions of the tracks, all suggested that these were pairs or groups of mesons created above ground in the air showers. Of the 37 events, 14 were coincident with discharges in one to 50 of the counters above ground. Only 2 of these coincidences were expected owing to chance, hence the observed association with air showers was 32 percent (including all directions of the mesons) instead of 1.5 percent as observed for the average events. If we assume that the zenith angle distribution was the same as that of single mesons, the method described above yields

TABLE IV. Numbers of recorded events, including all angles, classified according to the number of unshielded counters discharged on the surface of the ground.

Number of counters discharged, q	Observed events corrected for chance coincidences	Average number of electrons per event*	Number of shower electrons hitting counters	Range of density $\bar{\Delta}$, in m^{-2}	$dN/d \log_{10}\bar{\Delta}$
0	17,834	0	0	0–0.10	—
1	152^{+32}_{-25}	1.033	157	0.10–0.35	280^{+59}_{-46}
2	49±11	2.11	103	0.35–0.67	174±39
3	28±6	3.21	90	0.67–0.99	165±35
4	7	4.4	31		
5	2	5.5	11	0.99–2.35	40±10
6	2	6.6	13		
7	4	8.2	33		
8	2	9.2	18		
9	2	10.1	20		
10	1	11.5	11	2.35–4.6	38±12
11	3	14.9	45		
12	1	15.5	16		
13	2	16.5	33		
14	1	19	19		
16	1	21	21		
17	1	25	25	4.6 –8.9	17± 8
18	1	29	29		
20	1	31	31		
26	1	43	43		
39	1	65	65	8.9 –42	6±3
43	1	108	108		
50	1	160	160		
>50	0	...	...	42–∞	—
	18,098		1082		

* The probable number of electrons per event depended on the distribution among the 4 counter boxes; for example, 4 counters discharged in a single box, with zero in the other 3, indicated slightly more electrons than one counter discharged in each of the 4 boxes.

for $\bar{N}_2$, the average number of electrons in air showers associated with events in which at least two mesons are detected underground,

$$\bar{N}_2=(8\pm5)\times10^5.$$

$\bar{N}_2$ is 100 times as large as $\bar{N}$, the average number of electrons in showers associated with the detection of one or more mesons underground. The statistical error in $\bar{N}_2$ is very large, because $\frac{3}{4}$ of the detected electrons were recorded in only 2 of the events. However, it is clear that $\bar{N}_2\gg\bar{N}$, as would be expected according to our interpretation of the meson showers as originating in the air. It would not be expected if these events represented local penetrating showers created in the ground.††

The number of electrons in an air shower at sea level is the sum of contributions from high energy showers begun near the top of the atmosphere by a small number of photons resulting from π° decay, and showers of lesser energy begun at lower depths in the atmosphere by photons resulting from secondary interactions. The total energy in the electronic component must represent a large fraction, say one-half, of the energy of the primary. In the absence of a reliable theory of such complex showers, we assume all this energy to have come from the small number of photons in the first generation of the nuclear cascade. For showers of $\bar{N}=6800$ we assume the effective number of primary photons to be 10, and for $\bar{N}_2=8\times10^5$ we take the multiplicity to be 25.‡‡ The length through which the showers have developed is assumed to be 25 radiation lengths (900 g/cm^2). These assumptions yield for the average primary energies,

$$\bar{E}\simeq2.4\times10^{14}\text{ ev, and }\bar{E}_2\simeq9\times10^{15}\text{ ev.}$$

Again we emphasize that these values were computed from average numbers of particles, and hence are higher than median or typical energies; also that the assumptions made in applying the cascade theory cause uncertainty in the calculated primary energies.

Density Distribution of the Showers

Each air shower that was detected in coincidence with a particle underground could be associated with a mean density of electrons averaged over the area of the shower counters. This was first done crudely, on the basis of the number of counters missed and the number hit by the shower. Thus a rough relation between frequency and density was obtained, which seemed to follow a power law with exponent $-\beta\simeq-0.8$ in the integral distribution, except at densities less than 0.3 per m^2, where the exponent β seemed to become smaller, about 0.5.

By using the crude frequency-density relation thus derived, more accurate effective electron densities could be assigned to the showers discharging various numbers q of counters. Since the aim was to obtain the shape of the frequency-*vs*-density curve, the following definition of effective density $\bar{\Delta}_{\text{eff}}(q)$ was used. Let H(Δ) be the number of air showers associated with detected mesons and having density greater than Δ on the average over the counter surfaces. Then

$$H(\bar{\Delta}_{\text{eff}})=\int_0^\infty\left(-\frac{dH}{d\Delta}\right)P(\Delta,q)d\Delta=\text{const}\cdot(\bar{\Delta}_{\text{eff}})^{-\beta}.$$

$P(\Delta, q)$ is the probability for a shower of density Δ to discharge q or more counters. The exponent β is given one of the values determined by the preliminary analysis. Thus for each q an effective minimum density $\bar{\Delta}_{\text{eff}}(q)$ is determined, which is the density such that the total frequency of showers striking q or more counters equals the total frequency of showers having density greater than $\bar{\Delta}_{\text{eff}}$.

The steepness of the probability function $P(\Delta, q)$ made the determinations of $\bar{\Delta}_{\text{eff}}$ insensitive to the preliminary value of β except for $q=1$ and to a much lesser extent for $q=2$. The assumption of uniform average density over the widely separated counters leads to errors which are hard to evaluate but which cancel in first approximation. Even with this simplification the calculations of $\bar{\Delta}_{\text{eff}}$ were tedious and will not be given here.

The values of $\bar{\Delta}_{\text{eff}}$ and the corresponding frequencies are listed in Table IV and are plotted in the form of a differential density distribution in Fig. 14. The rectangles in this figure are consistent with a straight line, or power law with single exponent, except for the first rectangle, which represents the showers striking single counters. The departure of this point from the straight line is not so striking on the graph, but the height of this point depends on the assumed slope of the curve because of the influence of the slope on the calculations of $\bar{\Delta}_{\text{eff}}$ for showers striking single counters. If the density distribution were not allowed to become flatter in this region, the first rectangle would have to be wider and considerably lower, emphasizing the disagreement. Similarly, if the curve is flattened more than as shown, the first rectangle must be narrowed and raised. Thus, the change of slope is fairly well determined.

By assuming that all the points except the first one in Fig. 14 fit a straight line, and applying the method of least squares, the slope is calculated to be $\beta=0.93\pm0.12$. This at any rate is the average slope over the range of Δ from about $\frac{1}{3}$ to 30 particles per square meter.

†† As discussed in Section F, it is probable that half of the apparent pairs represent local showers, and only half of the events represent pairs of high energy mesons produced in the air. In that case, the value of $\bar{N}_2$, associated with the pairs of mesons created in air showers and detected at 1600 m.w.e., should be twice as large as the value given, and the corresponding energy $\bar{E}_2$ should be 1.6×10^{16} ev. The fraction of these events in which association with air showers above ground was detected would then be $\frac{2}{3}$ instead of $\frac{1}{3}$.

‡‡ These multiplicities were chosen because they are approximately the values predicted by Fermi's theory for primaries of the energies calculated. The results are fortunately not sensitive to the precise value guessed for the multiplicity.

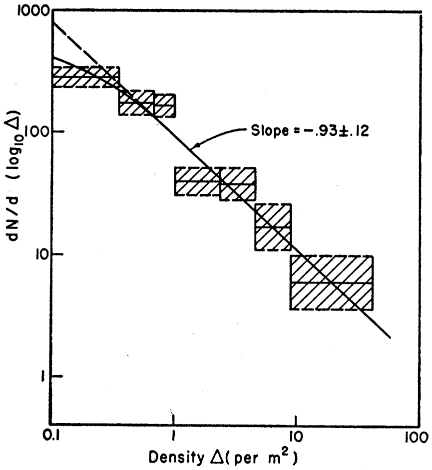

FIG. 14. Differential density distribution of the air showers associated with mesons 1600 m.w.e. underground.

At still higher densities the curve must gradually become steeper, else the average density and total number of electrons would diverge.

Multiplicity Distribution of the High Energy Mesons

Let N be the number of electrons in an air shower, and $G(N)$ represent the number of showers with more than N electrons. If $G(N)$ is represented by a power law, $G(N) \propto N^{-\beta}$, and if the lateral distribution of the electrons is insensitive to the number of them, the density distribution $H(\Delta)$ must also be a power law with the same exponent : $H(\Delta) \propto \Delta^{-\beta}$.§§ Therefore from the preceding discussion we infer that the showers associated with mesons detected underground have a number spectrum represented by a power law, with exponent $\beta = 0.93 \pm 0.12$ in the range of sizes contributing significantly to the coincidences among 2 or more shower counters (or to densities of $\frac{1}{3}$ to 30 electrons per m^2). Calculations show that these sizes are in the range 10^3 to 10^7 electrons, with greatest contributions around 10^4.

When the density spectrum of all showers is measured at the same elevation [24] without requiring detection of the mesons underground, a power law is again found, but with a different exponent, namely $\gamma = 1.35$ to 1.40 for the same range of shower densities. The difference between the exponents β and γ can be accounted for by the increase in probability of detection of a meson underground as the shower size increases. This probability is proportional to the number of high energy mesons in the showers (or for small showers, to the probability of having a single meson). Let the average number of mesons of range ≥ 1600 m.w.e. in a shower of N electrons be M. The fact that power laws fit the density spectra in both experiments suggests that M can be represented as proportional to a power of N: $M \propto N^{\alpha}$. The exponent α is required to be equal to $\gamma - \beta$. Therefore

$$M \propto N^{0.45 \pm 0.13}.$$

The flattening of the curve in Fig. 14 for small shower densities indicates that in small showers, when the average number M is only about one or even less, M is proportional to a higher power of N. Such an effect must be expected because of the high threshold for production of such energetic mesons.

The relation between M and N allows us to relate the multiplicity M with the primary energy E_0 of the shower. The number of electrons N is proportional to $E_0{}^s$ where s is the "age" parameter of the cascade theory. E_0 is assumed to be identical or proportional to the energy of the primary cosmic ray responsible for both the mesons and the cascade showers. The average value of s is taken to be 1.4 ± 0.05.|| || Thus we find

$$M \propto E_0{}^{0.63 \pm 0.18}.$$

The large value of the exponent in the relation between M and E_0 makes it clear that the mesons observed underground do not all originate in the initial nuclear interaction. The energy available for meson production in the center-of-mass system of the interaction is proportional to $E_0{}^{\frac{1}{2}}$, and the average energy of the secondaries in this system certainly increases with primary energy. Therefore in a single interaction the multiplicity depends on a power of E_0 less than $\frac{1}{2}$, more likely $\frac{1}{4}$ to $\frac{1}{3}$. The mesons observed underground must therefore sometimes originate by decay of π or other mesons created in the second or third generation of the nuclear cascade. On the other hand, if the number of generations in the production of the mesons were always large, the exponent would be about 1.0.¶¶ Since this is not the case, the number of generations involved must usually be small.

§§ See pp. 155–156 below, where the theorem is implicitly proved.

[24] G. Cocconi and V. Cocconi Tongiorgi, Phys. Rev. 75, 1058 (1949).

|| || In notes prepared for the Echo Lake Conference (1949) by one of us (KG) it was shown that the barometric effect of air showers, the altitude dependence, and the zenith angle dependence all indicated an effective age $s \simeq 1.27$. However, it is not clear that this age, which characterizes the energy spectrum of the particles, should be the same as the age that enters in the relation between N and E_0. The two values of s would be the same if air showers were single cascade showers initiated at one place in the atmosphere. But this is not so: energy is fed into the electronic cascade from the accompanying nuclear cascade as the shower proceeds through the atmosphere (see reference 36). Thus, the energy spectrum may exhibit a youth caused by new energy added to the shower far down in the atmosphere, but the major part of the total energy may have been dissipated in older parts of the shower, started near the top of the atmosphere. Cascade theory indicates that showers begun by 10–20 photons and containing 10^4–10^5 electrons after 25 radiation lengths have an "age" of about 1.45 to 1.5. The value $s = 1.4$ used above is the best compromise we know how to make in this complex situation represented by real air showers as contrasted with the idealized mathematical exercises of existing cascade theory.

¶¶ The approximate proportionality between M and E_0 for mesons of lower energy in extensive air showers has been inferred from measurements made above sea level. See Cocconi, Tongiorgi, and Greisen, Phys. Rev. 75, 1063 (1949).

From the relation between M and N and the known frequency of air showers as a function of N, the number of showers containing M high energy mesons can be calculated as a function of M. Let this function be $F(M)$.

$$F(M) \propto N^{-(\gamma+1)}dN/dM \propto M^{-(1+\gamma/0.45)} \propto M^{-4\pm1}.$$

The corresponding integral function, or number of showers with more than M mesons underground, would correspondingly be proportional to $M^{-3\pm1}$.

A second means of calculating $F(M)$ will be described in Section F, where the two results will be compared.

The probability of detecting two mesons underground in the same shower is approximately proportional to $M(M-1)$. By a simple extension of the discussion given above, we should expect the integral number spectrum $G(N)$ of the air showers associated with detected pairs of mesons to be very flat, going approximately as $N^{-0.5}$. Although the 12 showers recorded in association with 37 meson pairs are too small in number to provide a quantitative check of the shape of the distribution, they are at least in qualitative agreement with the predictions. Of these few showers, one had the highest density recorded in the whole experiment, another had the fourth highest density, and a third had the eighth highest density: thus 25 percent of them had densities among the uppermost 3 percent of all the showers recorded. Moreover, the percentage of the pairs which were associated with showers large enough to be detected was 23 times as high as the corresponding percentage of the mesons detected singly (or possibly 40–50 times as high; see footnote page 152).

Lateral Distribution of the Electrons in the Air Showers, and Minimum Number of Electrons

Figure 13 shows the fraction of the mesons for which associated air showers were detected, as a function of the mean horizontal distance from the shower counters to the path of the meson. For comparison, the curve drawn in the figure was calculated in the following way.

For each underground telescope, there is a projected resolution function $P(x, y)dxdy$ on the surface of the ground, representing the fraction of the mesons which struck the ground at coordinates x, y within area $dxdy$. Assuming no scattering of the mesons, this function is pyramidal in shape and 150 meters wide. We assumed that the number of associated shower cores hitting area $dxdy$ was represented by the same resolution function.

This amounts to assuming no scattering or angular divergence of the mesons from the cores of the showers. However, scattering over a width half that of the resolution function would not seriously affect the results of the calculations. The assumption is supported by the correlation, discussed above, between the apparent direction of the meson and the relative numbers of shower counters struck in the four boxes. Furthermore, the pairs of mesons detected underground decrease in number rapidly with increasing separation (see Fig. 16), indicating that the individual mesons of the pairs were on the average within about 10 meters of the "cores" of the events.

Let $F(N)dN$ be the differential number spectrum of the air showers associated with the mesons, and let $N\varphi(r)$ be the density of electrons at a distance r from the core of a shower containing N electrons. Then the fraction of the mesons, C, associated with showers striking at least one counter could be calculated from the equation

$$C=1-\int\int\int P(x, y)dxdyF(N)dN \times\exp[-NS_1(\phi_1+\phi_2+\phi_3+\phi_4)].$$

Here S_1 is the area of each of the four counter boxes, and $\varphi_1\cdots\varphi_4$ are the values of φ corresponding to the distances from the point x, y to each of the four counter boxes. Thus the separations of the boxes were taken into account.

The lateral distribution function $\varphi(r)$ which was used is the function calculated by Nishimura and Kamata[25] for cascade showers of age parameter $s=1.4$. The calculations were also made with the Molière function,[26] but the resulting curve was too steep to fit well the data in Fig. 13. This is not surprising, since the Molière distribution should apply at the maxima of showers ($s=1$) whereas most of the showers accompanying the mesons were small ones, well past their maxima. Therefore only the results obtained with the Nishimura-Kamata function are presented.

The number spectrum $F(N)dN$ is required to have the form $N^{-1.9}dN$ for large N, because of the density distribution shown in Fig. 14, derived from the relative frequencies with which different numbers of counters were discharged. A cutoff for low values of N was inserted at $N=N_{\min}$. The function $F(N)$ was normalized, implying that *all* the mesons detected underground were associated with air showers. Different values of $N_{\min}$ were tried, and the value which yielded the curve plotted in Fig. 13 was $N_{\min}=360$.

The integral of the above equation over N could be performed analytically, but the integral over x and y had to be done numerically for each of the counter telescopes underground.

The curve drawn through the calculated points fits the data in Fig. 13 very well; hence we must inquire how arbitrary were the assumptions used and how significant is the agreement.

Two of the assumptions have been justified already: the form of the number spectrum of the showers for large N, and the assumption that the mesons travel

[25] J. Nishimura and K. Kamata, Prog. Theor. Phys. **5**, 899 (1950); Prog. Theor. Phys. **6**, 628 (1951).

[26] G. Molière, in *Cosmic Radiation* (Dover Publications, New York, 1946).

close to the cores of the air showers. Once these assumptions are accepted, the form of the lateral distribution of the showers is required by the data on number of recorded showers *versus* distance of the counters from the shower cores. Therefore we may conclude that the cascade calculations of Nishimura and Kamata with age parameter $s=1.4$ are approximately correct at distances of 30 to 300 meters from the cores of the showers associated with mesons detected at 1600 m.w.e.

The agreement between the calculations and the observed number of showers, however, is not very sensitive to the precise value of the cut-off number $N_{\min}$, provided the number of large showers is held constant. That is, it is possible to assume that only a fraction of the mesons are associated with air showers and that these showers have a minimum size N^* greater than the value of $N_{\min}$ used in the calculations; but for $N>N^*$, the integral number spectrum $G(N)$ must still be given by $(N_{\min}/N)^{\beta}$ with the same values of $N_{\min}$ and β, 360 and 0.9. If N^* is made as large as 800, the reduction in the predicted number of small showers detected is about equal to the standard errors in the data. Therefore one can assume that as few as 50 percent of the mesons are associated with air showers provided these showers all have more than 800 particles. Our faith in the continuity of such relations, however, leads us to prefer the belief that the rest of the mesons are associated with smaller showers.

We may also inquire as to the maximum value of N for which we have evidence supporting the spectrum $G(N)$. The largest shower detected had a core about 150 meters from the counters, and discharged 26 of them, indicating a density of 10 particles/m^2. Since this was a large shower, probably near its maximum, we assume the Molière lateral distribution and thus find $N\simeq 1.3\times 10^7$ electrons. This shower was associated with *two* mesons detected simultaneously underground. Three other showers occurred (one of them associated with 3 mesons detected simultaneously underground), with cores approximately 250–290 meters from the counters, and each discharging 3 counters, indicating a mean density 0.8/m^2. These showers correspond to values of N around 5 to 10×10^6. One of these cases is likely to have been a chance coincidence, but this is compensated for by the fact that 30 percent of the mesons passed so far from the counters that associated showers of this size would have struck too few counters to allow an estimate of number of electrons. Thus there were approximately 4 showers of $N\geq 5\times 10^6$ in 18,100 events detected underground. The predicted number according to the spectrum given above is 3.5. In other words, the shape of the spectrum is determined by the data up to $N\simeq 10^7$ electrons.

Median Energy of the Primaries

We have concluded that half of the mesons detected at 1600 m.w.e. are associated with showers of more than about 800 electrons at sea level. The median energy of the primaries can therefore be estimated if we make assumptions about the fraction of the primary energy given to the soft component, the altitude at which this is done, and the multiplicity of photons or electrons at the origin of the shower. For these showers of comparatively small energy it seems reasonable to assume that most of the cascade energy originates in a small number of photons (e.g., 4) in the first generation of the nuclear cascade, at the 100-millibar level of the atmosphere; and that the cascade energy is half the total energy of the primary cosmic ray. Under these assumptions, the median energy is found to be 4×10^{13} ev.

From the fact that the single mesons have paths closer than 75 m to the cores of the air showers, we had previously estimated this energy (the lowest energy associated with detectable showers, which is approximately the same as the median energy of the primaries) to be greater than 2×10^{13} ev. Thus the two estimates are consistent.

Presence of High Energy Mesons in All Extensive Air Showers

It has been shown that the data are consistent with all the mesons detected at 1600 m.w.e. being contained in air showers of more than about 400 electrons. This could be so even if such air showers very rarely contained a meson of the required range. We now inquire what fraction of all air showers having more than 400 electrons seem also to contain such a meson.

The density spectrum of small air showers at Ithaca has been measured by Cocconi and Tongiorgi.[24] The exponent γ in the spectrum decreases slowly with the mean density, and for densities slightly less than 1 per m^2, γ seems to be about 1.25. γ is not likely to be lower than this, but may possibly be as high as 1.35. If we assume $\gamma=1.25$, the frequency of coincidences between counters of the largest areas, after correction by a factor 1.21 because of the counter separations, agrees with the integral density distribution $H(\Delta)$ given by

$$H(\Delta)=0.180\Delta^{-1.25} \text{ per sec}$$

with the density Δ expressed in m^{-2}.

It is well known that such a density spectrum is caused by showers having a number spectrum which is also a power law with the same exponent. Let $F(N)dN=K\gamma N^{-(\gamma+1)}dN$ be the frequency of showers containing N to $N+dN$ electrons. The contribution made by these to the showers of density $\geq\Delta$ at a detector is

$$\pi R^2(\Delta, N)\cdot K\gamma N^{-(\gamma+1)}dN$$

and the total frequency of showers with density $\geq\Delta$ will be

$$H(\Delta)=K\int_0^{\infty}\pi R^2(\Delta, N)\gamma N^{-(\gamma+1)}dN.$$

In these expressions we neglect the inclination of the showers relative to the vertical direction, because the zenith angle distribution is known to be very steep, being represented by $\cos^n\theta$ with $n \simeq 8$. $R(\Delta, N)$ is the distance from the axis at which a shower of N electrons has a density Δ. The geometry of the showers is assumed to be independent of shower size; therefore R depends only on the ratio $\Delta/N = r$, and the above equation may be transformed into

$$H(\Delta) = K\Delta^{-\gamma}\gamma \int_0^\infty \pi R^2(r) r^{\gamma-1} dr.$$

The integral in this equation was calculated numerically by use of the lateral distribution function given by Nishimura and Kamata[25] for cascades of age parameter $s=1.4$. By inserting the value of γ and the empirical expression for $H(\Delta)$ given above, the value of K was obtained, which completes the determination of the number spectrum $F(N)dN$. The integral number spectrum $G(N)$ or frequency of showers with cores striking unit area and having more than N electrons, can then be written

$$G(N) = \int_0^\infty F(N)dN = KN^{-\gamma}$$

with $K=4.3$ m^{-2} sec^{-1} and $\gamma=1.25$. $G(N)$ is the frequency of showers arriving in all directions. If the dependence of the rate on zenith angle θ is assumed proportional to $\cos^n\theta$ with $n \simeq 8$, the frequency per unit solid angle in the vertical direction is

$$G_v(N) = K'N^{-\gamma} \text{ m}^{-2} \text{ sec}^{-1} \text{ sterad}^{-1}$$

with $K'=6.1$ for $\gamma=1.25$. By inserting $N=400$ in this expression, we find the absolute vertical intensity at Ithaca of air showers containing more than 400 electrons:

$$G(400) = 3.4\times10^{-3} \text{ m}^{-2} \text{ sec}^{-1} \text{ sterad}^{-1}.$$

The calculations were repeated with $\gamma=1.35$. The result in this case is

$$G(400) = 5.3\times10^{-3} \text{ m}^{-2} \text{ sec}^{-1} \text{ sterad}^{-1}.$$

The absolute frequency of the mesons recorded underground was shown in Section B to be

$$I_v = 3.25\pm0.05\times10^{-3} \text{ m}^{-2} \text{ sec}^{-1} \text{ sterad}^{-1}.$$

However, the mesons do not occur singly. In the next section we shall infer from the coincidences of mesons underground that the average number of mesons per event is 1.2. Therefore the frequency of events containing mesons at 1600 m.w.e. is $I/1.2$ or 2.7×10^{-3} m^{-2} sec^{-1} sterad^{-1}. This is more than half of the frequency of showers containing more than 400 electrons on the surface of the ground. Within the accuracy of the calculation of $G(400)$, the rates are about equal. Slightly better agreement would appear to exist between the meson intensity and that of showers with more than 480 electrons (if $\gamma=1.25$) or 660 electrons (if $\gamma=1.35$), but the differences cannot be regarded as significant.

Since all the mesons detected at 1600 m.w.e. appear to be associated with air showers having more than about 400 electrons, and the frequency of all air showers with more than about 500 electrons is equal to the frequency of the mesons, we conclude that practically all air showers having more than 500 electrons contain a meson of range exceeding 1600 m.w.e. This picture is certainly over-simplified, in that there cannot be a sharp cut, at exactly any number of electrons, between showers that do and do not contain such a meson. The probability of a shower containing such a meson must be rising gradually with shower size in the neighborhood of 300–700 electrons, being large for showers of $N \gg 500$ and small for showers with $N \ll 500$.

F. MULTIPLE MESONS UNDERGROUND

Hodoscope pictures taken with the apparatus of Fig. 1 revealed, in addition to numerous soft showers and occasional penetrating and mixed showers, a small number of events in which two or three penetrating particles traversed the absorbers and counter trays in parallel straight lines. The probability of occurrence of such events by chance coincidences was completely negligible. These events differed from normal penetrating and mixed showers in the following respects:

(1) The number of associated penetrating particles detected was small, usually only two.

(2) The penetrating particles were usually not accompanied by secondaries incident on the apparatus from the rock overhead; and they failed to be absorbed or to be scattered or to produce showers in traversing the apparatus, which included almost a meter of lead and iron, 6.5 geometric mean free paths. Secondaries of ordinary penetrating showers have been shown to have almost geometric cross sections for further nuclear interactions.

(3) The tracks were parallel within the angular resolution of the apparatus. In some cases the parallelism could be established within 3 degrees for tracks separated by as much as half a meter. If these tracks came from an interaction in the rock, the secondaries must have traversed more than 10 meters of rock, or 40 geometric mean free paths. When normal penetrating showers contain a core of energetic particles collimated within a cone of small angle, the number of secondaries is very large.

In addition to these qualitative distinctions, the cases of parallel penetrating particles exhibited a remarkable correlation with air showers detected on the surface of the ground, as discussed in the previous Section. The high correlation indicates that the air showers were on the average much larger than those associated with single mesons underground.

All of these properties seem to fit best the assumption that the parallel penetrating particles are primarily μ-mesons created in the air showers by decay processes occurring in the atmosphere. It is well known that extensive air showers contain large numbers of μ-mesons, which have been detected by parallel tracks in cloud chambers or hodoscopes operated above ground and at small depths underground. It is reasonable to suppose that occasionally more than one μ-meson of very great energy is produced, and that when this happens the mesons follow nearly parallel paths close to the shower core. The probability of such an occurrence would increase rapidly with size of the air shower.

Coulomb scattering of mesons of about 10^{12} ev in traversing 1600 m.w.e. of ground, losing $\frac{2}{3}$ of their energy en route, would alter only their lateral positions by about $\frac{2}{3}$ meter. Therefore the Coulomb scattering alone would not prevent detection of mesons underground at distances less than a meter apart. The situation as regards nuclear scattering is not so clear. The cross section measured by Amaldi and Fidecaro[27] is so small that if correct, almost all the mesons should escape nuclear scattering; but the observations of Nash[28] and of Trent[29] yield cross sections such that almost all the mesons should undergo large angle scattering several times in traversing 1600 m.w.e. of ground. The Lorentz factor of the center-of-mass system when a meson of 7×10^{11} ev collides with a nucleon is about 20; hence each scattering should produce, on the average, about 15 meters lateral displacement of the mesons detected 600 meters underground. However, in view of the discrepancy between the above experiments, the importance of nuclear scattering is doubtful. It seems significant that Hazen[30] did not report any observations of large angle scattering of mesons 850 m.w.e. underground, while the mesons traversed a total path length of 100 meters Pb in crossing the plates within a cloud chamber.

The number of events in which parallel penetrating particles were seen with the apparatus of Fig. 1 is 43, while in the same time 21,070 single mesons were recorded, yielding a ratio of 2 coincidences per 1000 single mesons. Four of the events, or 1 per 5000 single mesons, showed three parallel tracks, while the others showed two.

These numbers of events are uncertain, however, because the mesons at 1600 m.w.e. underground frequently produce soft showers, and when 2 or 3 mesons traverse the apparatus it is highly probable that electron secondaries should confuse the picture and make the event resemble either a normal penetrating shower or (when the tracks are close together) a single meson with soft secondaries. One-third of the 43 events were sufficiently complex that the identification was in some doubt, while an even larger number of other events were possible cases of parallel particles but were not so classified because of doubtful appearance.

In order to investigate the phenomenon further, five counter arrangements like the one in Fig. 7 were set up and operated simultaneously. Simple coincidences between the two counter trays of each set were recorded only with a mechanical register, but when such coincidences occurred simultaneously in two counter sets or when two or more counters were hit in each tray of one set, a hodoscope picture was taken, permitting identification of each counter discharged. The five counter sets were oriented with counters parallel to each other. Thus parallel particles traversing separate counter sets could be recognized, as well as parallel particles within a single set provided the particles were sufficiently separated to prevent confusion with soft showers.

For 1863 running-hours the five counter sets were arranged as in Fig. 15(a),*** such that there were ten different spacings between pairs, ranging from 1.5 to 17.5 meters. Following this run, for 1940 hours the sets were arranged as in Fig. 15(b), so as to provide more information for pair spacings between 10 and 23 meters. The roof of the passage in the mine was one to two meters above the counters, a distance small compared with the larger separations. All the trays were protected by lead against soft showers from the roof. In the second counter arrangement, furthermore, a wall of rock several meters thick was situated between one of the sets and two of the remaining four. Therefore, when particles were detected simultaneously in separated counter sets, there was no ambiguity, as there was at small spacings, in distinguishing the parallel particle events from single mesons accompanied by soft showers, or in distinguishing them from normal penetrating showers.

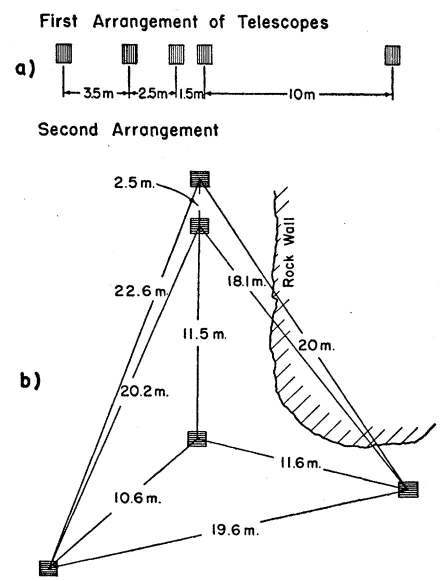

FIG. 15. Two arrangements, viewed from above, of the counter telescopes used in the study of the meson showers occurring at 1600 m.w.e. Each telescope is like the one shown in Fig. 7. The lines indicate the direction of the axes of the counters.

*** One of the five sets was inactive for 241 hours of this running time.

[27] E. Amaldi and G. Fidecaro, Phys. Rev. **81**, 339 (1951).

[28] F. Nash, Manchester University thesis (1951), reported by E. P. George in *Progress in Cosmic Ray Physics* (Interscience Publishers, Inc., New York, 1951).

[29] P. T. Trent, London University thesis (1951), reported by E. P. George in *Progress in Cosmic Ray Physics* (Interscience Publishers, Inc., New York, 1951).

[30] W. E. Hazen, Phys. Rev. **86**, 764 (1952).

Parallelism of the Penetrating Particles

The counters were of 2-in. diameter with a vertical separation of 7 in.; therefore the angular resolution for each particle was $\pm 16°$ when the tracks were nearly vertical and no soft secondaries were present, which was true in the majority of the cases. Thus in a single event the angle between the tracks could only be known within $\pm 32°$. However, the *average* angle between the tracks could be much more accurately determined.

When two perfectly parallel particles were detected, the two lines joining the axes of the discharged counters may be parallel, or may be 16° convergent or 16° divergent. The average probabilities of the three orientations are $\frac{2}{3}$ parallel, $\frac{1}{6}$ convergent, and $\frac{1}{6}$ divergent. If the pairs of tracks are consistently divergent even by an angle small in comparison to the angular resolution of the detectors, the relative probability of convergent and divergent lines joining the counter axes is strongly altered. For instance, if the angle is $\tan^{-1} 0.031$, the ratio is reduced by a factor 2.

In Table V are summarized the data on parallelism of the observed pairs of tracks. For this tabulation we have considered only the events in which no extra counters were discharged by soft secondaries of the mesons, and only the pairs occurring in separated counter sets. It is observed that the ratio of apparently convergent to apparently divergent pairs is greater than one, rather than less than one. Because of the poor statistics the data may be consistent with a one-to-one ratio, but they cannot be consistent with an average divergent angle as large as $\tan^{-1} 0.02$. If one imagines the tracks as due to production of penetrating pairs which travel unscattered through the rock, the pair separations of ten meters and more therefore require the origins to be 500 meters or more above the apparatus. Since the surface of the ground was 600 meters above the counter sets, such interactions occurring in the ground may be ruled out.

If, instead, the pairs are imagined to represent secondaries produced in the ground but scattered through angles greater than the angular separation in the original interaction, scattering should roughly equalize the numbers of apparently convergent and divergent pairs, but should reduce the number of apparently parallel pairs below the fraction $\frac{2}{3}$ expected for strictly parallel particles. The experimental fraction is in good agreement with $\frac{2}{3}$ at all separations, and is greater than about 0.55 even at separations larger than 10 meters. This requires the average angle between the tracks to be less than $\tan^{-1} 0.1$. Since the scattering occurs along the tracks rather than at the origin, the origins must therefore be 200 meters or more above the depth of the apparatus. Showers with such ranges, occurring in condensed matter, are entirely unknown; therefore it is more likely that the pairs represent high energy μ-mesons created in the air in extensive air showers.

TABLE V. Parallelism of pairs of tracks.

Distance between pairs	≥ 1 m	≥ 6 m	≥ 10 m	≥ 14 m
No. of pairs	118	65	38	14
No. apparently parallel	78	40	24	9
No. apparently diverging	14	8	5	1
No. apparently converging	26	17	9	4
Fraction apparently parallel	0.66 ± 0.04	0.62 ± 0.06	0.63 ± 0.08	0.64 ± 0.13
Ratio Div./Conv.	0.54 ± 0.18	0.47 ± 0.20	0.56 ± 0.31	$0.25 {+0.3 \atop -0.25}$
Expected number converging and diverging if all are really parallel	20	11	6	2

Lateral Distribution

The decoherence curve of meson showers, or frequency of pairs of mesons as a function of distance between them, can provide information about the numbers of high energy mesons in the air showers and the lateral distribution of the mesons. Let $F(M)d\sigma$ be the number of showers containing M mesons underground, and having cores that strike a small area $d\sigma$ at distances r_1, r_2 from two detectors. Let A_1 and A_2 be the areas of the detectors and assume these areas small compared with the area over which the mesons are distributed. Let x be the separation between the detectors, and let $A\varphi(M, r)$ be the probability that a given meson in a shower of M mesons falls in an area A at a distance r from the core at $d\sigma$. These definitions are quite general. Then the number of coincident pairs recorded at distance x, $C_{12}(x)$, is given by

$$C_{12}(x)=A_1A_2\sum_M M(M-1)F(M)\int \varphi(M,r_1)\varphi(M,r_2)\,d\sigma,$$

the integral being carried out over the horizontal plane.††† Now multiply $C(x)$ by $2\pi x dx$ and perform a second integral over the horizontal plane. On the right-hand side, the integral over x may be performed first, holding the positions of $d\sigma$ and A_1 constant and moving the position of A_2, as indicated by

$$\int 2\pi x C_{12}(x)dx=A_1A_2\sum_M M(M-1)F(M) \times\int \varphi(M, r_1)d\sigma\int \varphi(M, r_2)2\pi x dx.$$

The first integral equals 1 by definition, since the integral extends over the entire plane, whereupon the second integral equals 1 also, and the result becomes

$$\int 2\pi x C_{12}(x)dx=I_0A_1A_2\langle M(M-1)\rangle_{\text{Av}},$$

††† This equation assumes that the probability of recording three mesons is small compared with that of recording two; i.e., that $MA\varphi \ll 1$. We shall see that the distribution function $F(M)$ is so steep that most of the twofold coincidences are detected in showers for which this assumption is good; i.e., the number of threefold coincidences detected was less than 10 percent of the number of pairs.

where I_0 is the total number of showers per unit area, and $\langle M(M-1)\rangle_{Av}$ is the average value of $M(M-1)$.

By a similar but simpler process we find that the number of single mesons N_1 recorded in one of the detectors, say A_1, in the same time is

$$N_1 = I_0 A_1 \langle M\rangle_{Av}.$$

Therefore

$$\int 2\pi x \frac{C_{12}(x)}{N_1 A_2} dx = \frac{\langle M(M-1)\rangle_{Av}}{\langle M\rangle_{Av}} = \frac{\langle M^2\rangle_{Av}}{\langle M\rangle_{Av}} - 1.$$

The experimental data on numbers of coincidences $C_{12}(x)$ as a function of x are presented in Fig. 16. The counters in the five detectors were not exactly alike in size and the running times for pairs of detectors at different distances x were not all equal, but the conditions at different distances x are equalized by presenting the results in the form of the ratio $C_{12}(x)/N_1A_2$, which is the same as $C_{12}(x)/N_2A_1$ and is directly applicable in the equation above. For equal detectors of unit area, the quantity is the ratio of the coincidence rate to the single counting rate.

For the points in Fig. 16 at distances less than one meter, the ordinates were obtained from the events in which two parallel penetrating particles traversed a single counter set. In counting these events it was necessary to adopt some selection procedure which made it unlikely for single mesons accompanied by soft showers to be counted as a pair of mesons. Two procedures were tried, which gave equal results, as indicated by the two points in Fig. 16 at 0.5 and 0.6 meter separation. The first method was to count a coincidence whenever two particles crossed a single counter set in possibly parallel directions and far enough apart to leave at least two counters undischarged between them in both counter rows. To normalize these coincidences to those measured between separate counter sets the following factors had to be applied: (1) a factor $\frac{1}{5}$ because there were 5 counter sets in which such coincidences were counted, (2) a factor 2 because the ways of permuting M particles so that two of them land in a given box are half the permutations that place one particle in each of two boxes, and (3) a factor 1.61, because of the reduction in effective area for the second particle by the requirement of undischarged counters. This factor was determined from a study of the zenith angle distribution and the numbers of counters hit by soft secondaries of single mesons. The second method was to divide each counter set into 3 adjacent sets each 5 counters wide, and to tabulate only the coincidences between the extreme sets. In this case the correction for the reduction in effective area was determined to be $(3.6)^2$.

The function $2\pi x C_{12}(x)/N_1A_2$ is plotted in Fig. 17 and shows that although the decoherence curve (Fig. 16) falls off continuously with increasing separation, most of the pairs are to be found at distances on the order of 10 meters or more. This is a further argument in favor of the interpretation that these are pairs of mesons arising in extensive air showers rather than created by local interactions in the earth.

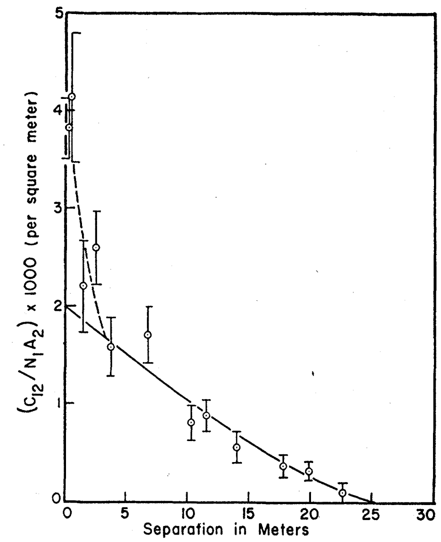

FIG. 16. Decoherence curve of the meson showers. C_{12} is the number of coincidences of parallel particles in two counter sets, N_1 is the total number of penetrating particles recorded in counter set no. 1, and A_2 is the area of counter set no. 2 ($N_1A_2 = N_2A_1$). The solid curve is calculated on the assumption that the lateral distribution of mesons about a shower core is flat out to a radius of 13 meters and zero at larger radii.

The area under the smooth curve in Fig. 17 is about 1.1, though the possible existence of a longer tail may make the value slightly higher.‡‡‡ Thus we have $\langle M^2\rangle_{Av}/\langle M\rangle_{Av} = 2.1$ and since $1 < \langle M\rangle_{Av} < \langle M^2\rangle_{Av}^{\frac{1}{2}}$ one may infer the inequalities $2.1 < \langle M^2\rangle_{Av} < 4.4$. Thus the showers have on the average very few mesons at 1600 m.w.e. underground, and have a number distribution $F(M)$ that decreases with increasing M more rapidly than M^{-3}. If a power law is assumed for the number distribution, the exponent is sensitive to the ratio $\langle M^2\rangle_{Av}/\langle M\rangle_{Av}$, and must be about 3.45. In this case $\langle M\rangle_{Av} = 1.20$ and $\langle M^2\rangle_{Av} = 2.5$.

From the above observations we may derive still another argument to the effect that the particles detected in these events are not local secondaries created in the ground. If they were local secondaries, since they are numerous (approximately one in every 6 mesons) and since they appear far enough apart to be detected singly, they should give rise to a measurable local absorption greater than that expected for μ-mesons originating entirely in the air. In part A of this report it was shown that the excess absorption was 2 ± 2 per 1000 penetrating particles in 363 g/cm² of rock. If this is entirely due to the particles appearing

‡‡‡ The observations on the associated electronic air showers, discussed in the previous Section, indicated that few if any of the mesons were as far as 75 m from the axis of the event. Therefore the tail of the distribution cannot contain a very large number of particles.

in the penetrating pairs observed, the absorption of the secondaries in 363 g/cm^{-2} of rock must be only 1.2 ± 1.2 percent, about as small as the absorption of the μ-mesons which have penetrated all the rock overhead (0.8 percent). Since the mean range of these "secondaries" must be about as great as that of the primary penetrating component, both components probably originate in the atmosphere and are identical in nature.

Narrow Showers

The decoherence curve in Fig. 16 exhibits a sharp rise as the separation of the tracks is reduced below 2 or 3 meters. The increase stands out clearly in the two points obtained from pairs seen within single counter sets. In addition to these two points, a third may be derived from the observations made with the counter arrangement of Fig. 1. Forty-three events showing two or more parallel penetrating particles were seen among 21,160 recorded mesons. The sensitive area for detecting the second member of a pair was about 0.8 m^2. Multiplying by 2 for comparison with coincidences in separate detectors, we obtain

$$\frac{\text{pairs}}{\text{singles}}\times\frac{2}{A}=\frac{43}{21{,}160}\times\frac{2}{0.8}=5\times10^{-3}\text{ per m}^2$$

with an uncertainty of about 30 percent due to difficulty in identifying some of the events. This ratio agrees with the two points plotted in Fig. 16 in indicating a large change in frequency of pairs as the separation varies from $\frac{1}{2}$ m to 2 or 3 m.

The steepness of the decoherence curve in this region implies a lateral distribution function having a sharp peak at distances within about one meter of the shower axis. This is not consistent with the hypothesis that all the penetrating particles originate in the air in extensive showers. The Coulomb scattering in the ground alone would flatten the distribution over a radius of about $\frac{2}{3}$ meters from the core; i.e., for particle separations up to about 1 meter. Even more convincing is the observation that, neglecting scattering, one meter separation underground corresponds to an angle of less than 10^{-4} radian at the altitude where the high energy mesons are created in air showers. Collimation of the mesons in such a narrow cone would imply primary energies around 10^{17} ev, which would produce much larger air showers than were observed.

Therefore we conclude that the sharp rise in the decoherence curve at distances less than a few meters is not due to mesons generated in the air, but to local secondaries produced in the ground, and therefore found much closer together. In accordance with this interpretation, the decoherence curve drawn in Fig. 16 is represented as the sum of two curves, of which the narrower represents the local interactions. The number of pairs involved in the narrow part of the decoherence curve can be estimated by multiplying by $2\pi xdx$ and integrating over x, the separation of the detectors. As can be seen in Fig. 17, this integral must be very small. It is approximately 0.018, less than 2 percent of the integral over the broad part of the decoherence curve.

The number of secondary particles required to explain the narrow portion of the decoherence curve is less than one percent of the total number of penetrating particles; and the distances of the secondaries from the meson that has penetrated underground are so small that only a fraction of the secondaries would be detected as single particles. Therefore the effect of the secondaries on the local absorption of single penetrating particles measured underground would be small. In a thick absorber, such as the 20-in. Pb which we used, the increase in the number of particles stopped can be estimated to be on the order of 2 per 1000 mesons. The excess absorption found in Part A of this report was 2 ± 2 per 1000 mesons, sufficient to permit the presence of these secondaries.

The local showers which appear as pairs of parallel penetrating particles must represent only a small fraction of all the local penetrating showers, because of the selection criteria employed. Most of the penetrating showers are complex events including numerous secondary particles with nonparallel directions. The cross section for production of penetrating showers by μ-mesons is discussed in Section G.

Energy of the Primaries

The broad portion of the decoherence curve in Fig. 16 is to be interpreted in terms of the lateral and angular distribution of μ-mesons of mean energy 10^{12} ev in extensive air showers. It is well known that the shape of a decoherence curve is not very sensitive to the precise shape of the lateral distribution function of the particles; therefore we employ the simplest function which fits the data, and only attempt to find the average or characteristic width of the lateral distribution. The curve drawn in Fig. 16, which happens to fit the data rather well, corresponds to a uniform distribution of mesons within a distance σ from the shower axis; i.e.,

$$\varphi(M,r)=\begin{cases}1/\pi\sigma^2 & \text{for } r<\sigma\\ 0 & \text{for } r>\sigma\end{cases}$$

with σ taken as 13 meters.§§§ This function implies a mean distance of the mesons from the shower axis, $\bar{r}$, equal to 8.7 m, and a root mean square distance $\langle r^2\rangle_{\text{Av}}^{\frac{1}{2}}$ equal to 9.2 m. The calculated decoherence curve is

$$C_{12}(x)=C_{12}(0)[1-2/\pi\sin^{-1}(x/2\sigma)-x(4\sigma^2-x^2)^{\frac{1}{2}}/2\pi\sigma^2].$$

A Gaussian distribution function can be applied with equal simplicity, and yields a decoherence curve which is also Gaussian in shape. With a single width parameter σ, this does not fit the data as well as the square

§§§ $\varphi(M,r)$ is the probability that a given meson in a shower of M mesons falls in unit area at a distance r from the shower axis.

lateral distribution. If individual showers have Gaussian distributions of particles, but the root mean square width varies over a small range from one shower to another, there are plenty of free parameters to fit the observed distribution. However, it seemed that little could be learned by carrying out the fitting process.

There are reasons for believing that the mean width of the showers does not vary by a large factor from one shower to another. Aside from purely statistical fluctuations, the expected causes of variation in lateral spread are three: variation in primary energy of the interactions in which the mesons are produced (π or κ) that decay into μ-mesons, variation in impact parameter of the nuclear interaction, and variation of the altitude at which the interactions occur. The primary energy has a minimum value set by the necessity of producing at least two μ-mesons that can penetrate 1600 m.w.e. of ground; this may usually require the production of 10 or more high energy π- or κ-mesons. When the primary energy is much higher than the minimum value, not only is the frequency of occurrence reduced, but the probability of decay of the secondary mesons into μ-mesons diminishes. Ultimately an energy is reached such that the mesons arise from decay of tertiaries produced in nuclear interactions of the secondaries, instead of by decay of the secondaries produced in the initial reaction. Still further increase in primary energy causes the μ-mesons to be produced in a later generation. Thus, the energy which determines the lateral spread of most of the μ-mesons in a shower at a given depth underground does not increase indefinitely with primary energy. And the lateral spread depends only on the square root of the energy of the interacting particle.

According to the Fermi theory,[31] as the impact parameter increases, the width of showers of a given primary energy goes down, but the number of secondaries also diminishes. Therefore, the primary energy required to produce secondary mesons capable of penetrating underground decreases with increasing impact parameter. Thus, showers in which secondary mesons of a given energy are produced do not vary markedly in width. Moreover, collisions occurring with very large impact parameter, which would result in narrow showers, are not very effective in producing coincidences of mesons underground, because of the low multiplicity of the secondaries. In the larger showers, wherein the μ-mesons observed underground are produced in numerous second or third generation interactions, there is an averaging over impact parameters in determining the width of single showers, making them all rather alike.

As for the altitude variation, the exponential change of pressure with altitude leaves very little atmosphere for interactions to occur in above 30,000 meters, while nuclear absorption and the improbability of decay in the high pressure region of the atmosphere prevent the origin of significant numbers of the showers below about 8000 meters. It is therefore a reasonable approximation to assume that all the showers originate at an altitude around 16,000 meters, which is the altitude of the 100 millibar level.

The decoherence curve of meson showers underground indicates, regardless of the type of function used to fit it, that the density of the mesons is large only within 13 meters of the axes of the events. Since Coulomb scattering and the angles introduced in the μ-decay process (of either π- or κ-mesons) account for only a small fraction of this width, we assume the width is entirely due to the angular distribution of secondaries in the nuclear interactions in which the mesons are produced. By assuming that the interactions occur at 16 kilometers, we find that most of the secondaries are within an angle of 0.8×10^{-3} radian.

By using the Fermi theory, averaging over impact parameters with a weighting factor depending on the number of secondaries produced, and neglecting the mesons emitted backward in the center-of-mass system (because of their low energy in the laboratory frame of reference), we find that the angular distribution can be approximated as rectangular with a width $1/(3\bar{\gamma})$, $\bar{\gamma}$ being the Lorentz factor in the relativistic transformation. Assuming the interaction to be a nucleon-nucleon collision, $\bar{\gamma}$ equals $(E_0/2Mc^2)^{\frac{1}{2}}$, with E_0 the primary energy and Mc^2 the rest energy of a nucleon. By equating $1/(3\bar{\gamma})$ to the angular width 0.8×10^{-3} radian, we obtain $E_0\simeq3\times10^{14}$ ev as the effective primary energy in the production of pairs of mesons detected at 1600 m.w.e. underground.

In Section E, the data on the air showers associated with pairs of mesons detected underground were dis-

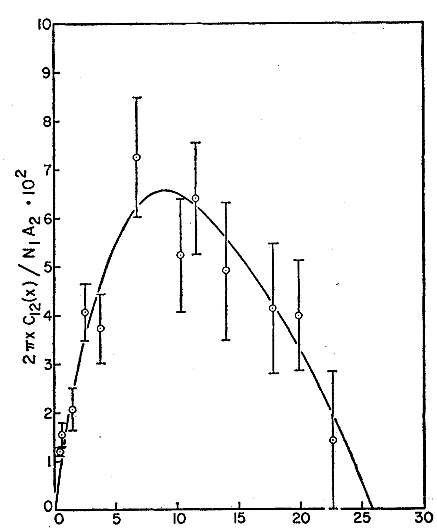

FIG. 17. Frequency of coincidences of parallel penetrating particles times separation of the detectors (and multiplied by a normalizing factor), plotted against the separation. The area under the curve determines the mean square number of mesons per shower relative to the mean number of mesons per shower.

[31] E. Fermi, Prog. Theor. Phys. 5, 570 (1950); Phys. Rev. 81, 683 (1951).

cussed, and it was noted that the fraction of the pairs coincident with observed air showers was 20 to 40 times as large as the fraction of single mesons. This requires the median size of the air showers associated with pairs of mesons seen underground to be about 6×10^4 to 2×10^5 electrons. If the multiplicity of photons at the origin of such a shower is about 20, the energy in the soft component is about 6 to 16×10^{14} ev. This is assumed to be half of the total primary energy of the particle initiating the event; therefore the median primary energy is estimated as 1 to 3×10^{15} ev. The upper figure corresponds to assuming that half of the pairs seen at spacings less than 1 meter are due to local penetrating showers and the other half to pairs of mesons coming from the air; and that the local showers have about as little correlation with observed air showers as the mesons detected singly. In accordance with the discussion on p. 160, the more likely median energy is therefore 3×10^{15} ev.

The difference between the two estimates of primary energy is reasonable since the energy E_0 inferred from the width of the decoherence curve is not necessarily the total primary energy, but the energy of the particles causing the interactions in which the mesons seen underground originate. The energy of the air showers should correspond to E_0 if the mesons are produced in the first interaction, but should be larger than E_0 if the mesons are created in the second or third generation of the nuclear cascade.

The ratio of pairs to single mesons detected and the ratio of threefold coincidences to pairs (discussed below) indicate that the detected pairs are contained in showers in which the median number of μ-mesons underground is about 10. In view of the improbability of $\pi-\mu$ decay at high energy, the number of high energy pions produced in such a typical event is likely to be 100 or more. It is therefore to be expected that some of the mesons are products of the second generation, and that the angular distribution should correspond to an energy an order of magnitude less than the total primary energy.

A third means of estimating the median primary energy of the pairs seen at 1600 m.w.e. is given by the observation, mentioned above, that the median event is one which contains about 10 mesons. The primary energy must increase somewhat more rapidly than the number of mesons, because the multiplicity in single interactions is a slowly increasing function of primary energy, and the probability of π-μ or κ-μ decay decreases with increasing energy. The analysis of air shower data in Section E and the analysis given on pp. 163–164 indicate that the number of mesons at 1600 m.w.e. increases with approximately the 0.7 power of the primary energy. Therefore an event containing 10 mesons is due on the average to a primary of energy about 30 times greater than that of the primaries creating single mesons reaching the same depth. The median energy of the primaries of single mesons was estimated to be 4×10^{13} ev; multiplying by 30 yields 10^{15} ev, in reasonable agreement with the other estimates made above.‖‖‖

Multiplicity Spectrum of the Meson Showers

In addition to the coincidences of two parallel penetrating particles underground, there were a small number of threefold coincidences and even of events containing more than three parallel particles. In order to prevent the inclusion of local penetrating showers created in the ground, we have analyzed only those threefold coincidences involving at least two separated counter sets of the type shown in Fig. 7. Each disposition of the five counter sets provided 10 pairs of sets, in which a triple coincidence could be observed by having two particles in one set and one in the other; and 10 ways of selecting three counter sets in which a triple coincidence could be recorded if one particle appeared in each set. These possible combinations were classified according to the maximum distance required between the particles. The numbers of threefold coincidences at these distances were tabulated, as well as the numbers of twofold coincidences registered in an equivalent time with detectors of the equivalent areas at the same separations.

No significant variation with distance was seen in the relative frequency of twofold and threefold coincidences, but this may have been due to the poor statistics, since the total number of threefold coincidences was only 19, occurring in 16 independent events. (Four threefold coincidences were accounted for by one event in which 4 separate counter sets all showed parallel penetrating particles.) By summing over all the possible combinations, the average ratio of threefold to twofold coincidences was found to be 0.069 ± 0.018, or about one in 15.

This ratio, and the ratio of twofold coincidences at zero separation to single mesons recorded, are both very sensitive to the form of the function $F(M)$ giving the relative frequencies with which showers of M parallel mesons occur underground. If we assume $F(M)$ has the form of a power law, $M^{-\delta}$ (for integral values of M), either ratio can be used to determine δ, and the equivalence of the two determinations can be used to verify that the power law is a fairly accurate representation of $F(M)$.

‖‖‖ At the median primary energy for production of the mesons detected singly, the probability of producing no mesons that reach 1600 m.w.e. is probably appreciable, and the average number of mesons reaching 1600 m.w.e., produced by a primary of the median energy, would then be slightly less than one. However, the data on the air showers indicate that the transition between producing and failing to produce such mesons is a rather sharp function of primary energy; hence at the median primary energy the average number of the mesons cannot be very much less than one. Assuming that this average is 0.7 instead of 1.0, the ratio of median energies for producing mesons detected singly and in pairs becomes $(10/0.7)^{1/0.7}=45$ instead of 30, yielding a value close to 2×10^{15} ev for the median energy of the primaries of the meson showers observed at 1600 m.w.e. This is probably more accurate than the value 1×10^{15} ev given above, but the difference is within the estimated uncertainty of the result, which may be stated as $E_0=(2\pm1)\times10^{15}$ ev.

Assume the M mesons in a shower are distributed uniformly within a circle of radius σ (as assumed above in calculating the decoherence curve shown in Fig. 16); then the relative frequency of twofold and threefold coincidences will be independent of the separation of the counter sets, in accord with our rough observations. Let $p=A/\pi\sigma^2$ be the probability that a given particle falls in a counter set of area A within the circle of radius σ. Then the exact probability of having one or more of the M particles in each of two specific detectors is $1-2(1-p)^M+(1-2p)^M$;¶¶¶ the probability of having one or more particles in each of three counter sets is $1-3(1-p)^M+3(1-2p)^M-(1-3p)^M$; and the probability of one or more particles striking a single counter set is $1-(1-p)^M$. By multiplying these probabilities with $M^{-\delta}$ and summing over M, one may find the expected ratio of threefold to twofold coincidences and the ratio of twofold coincidences at zero separation to single mesons recorded. This has been done approximately for values of δ in the range 3 to 4, with the results shown in Fig. 18. The value of p used in these calculations is 1.26×10^{-3}, corresponding to $\sigma=13$ m, as indicated by the decoherence curve, and corresponding to an area $A=0.67$ m^2, which is the average value corrected for the zenith angle dependence of the mesons and for the influence of soft secondaries.

The experimental ratios of threefold to twofold and twofold to single coincidences are shown on the graph in Fig. 18, the latter ratio being calculated from the intercept of the extrapolated decoherence curve supposedly excluding local penetrating showers (see Fig. 16 and discussion above).**** Both the ratios indicate values of δ very close to 3.4, thereby verifying that a power law is a reasonable approximation to the number distribution $F(M)$, and determining the exponent with an uncertainty of about 0.1. The ratio of the number of pairs of parallel particles to the number of single mesons detected depends principally on the shape of the function $F(M)$ for small values of M, single mesons being detected mostly in the showers containing only one or two mesons, and pairs being detected mostly in showers of 2 to 30 mesons. The ratio of double to triple coincidences, on the other hand, depends principally on the shape of $F(M)$ for larger values of M, since a large fraction of the triple coincidences must have been due to showers containing more than 100 mesons underground.

In Section E the observations on the air showers were also shown to lead to an exponential number spectrum for $F(M)$, with an exponent $\delta=4\pm1$ derived from the density distribution of the extensive air showers associated with the mesons underground. The two independent deductions are thus consistent with each other, but the exponent derived from the multiple coincidences underground is more precise than the one derived from the air showers. We may use the new value of δ, therefore, to improve the relations derived in Section E between the number of mesons capable of penetrating 1600 m.w.e., M, and the number of electrons

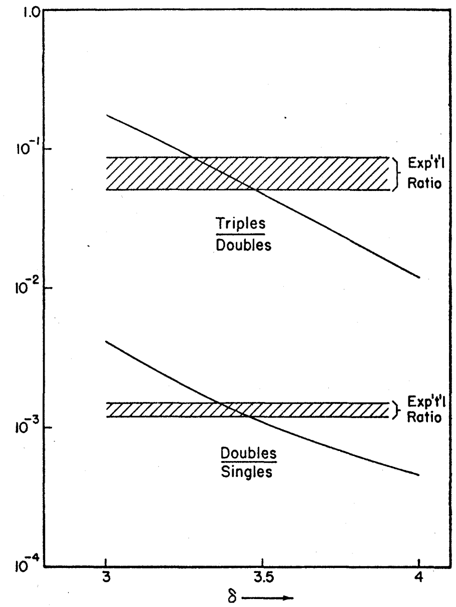

FIG. 18. Smooth curves give the calculated ratio of triple to double coincidences of parallel penetrating particles, and the calculated ratio of double coincidences to single mesons detected, as functions of the exponent δ in an assumed power law representing the multiplicity distribution of mesons in the showers. The lateral distribution of the mesons is assumed flat out to 13 meters from the core and zero thereafter, as in Fig. 16. The shaded regions give the experimental ratios.

in the air shower at sea level, N, or the primary energy of the shower, E_0.

Since the integral number spectrum of the electrons in air showers goes as $N^{-\gamma}$ with $\gamma=1.4\pm0.1$ over the range of N with which we are concerned, and the differential number spectrum of the highly penetrating mesons in the showers goes as $M^{-\delta}$ with $\delta=3.4\pm0.1$, we have:

$$M\propto N^{1.4/2.4}=N^{0.58\pm0.05}$$

as compared with the previous expression, $M\propto N^{0.45\pm0.13}$. The present treatment includes air showers of large size; hence we assume for the relation between number of electrons and primary energy, $N\propto E_0^{1.3\pm0.1}$. This

¶¶¶ In the approximate treatment given on page 158 we approximated this probability as $M(M-1)p^2$, which is accurate for $Mp\ll1$. Since p is about 1.26×10^{-3}, the condition is fulfilled for $M\ll800$ mesons. The large ratio between single and twofold coincidences requires the exponent δ to be in the range 3.4–3.5. Correspondingly, the median value of M for the showers producing twofold coincidences is in the range 10–13, and very few of the showers fail to fulfill the required inequality. The threefold coincidences, on the contrary, are frequently caused by large showers for which the approximation $Mp\ll1$ is not valid, but the approximation $(1-np)^M=\exp(-npM)$ may be used with little error in this case.

**** The ambiguity arising from the form of the decoherence curve at small distances is very little, since the shape of the function $F(M)$ is determined primarily by the integral of $xC_{12}(x)$ rather than by $C_{12}(0)$.

yields, for the dependence of M on E_0,

$$M \propto E_0^{0.75\pm0.09}$$

as compared with the former expression $M \propto E_0^{0.63\pm0.18}$. The new relations are in sufficient agreement with the old ones that the qualitative discussion of their significance, given in Section E, needs no modification.

Core Structure of the Air Showers

The present study of the showers of mesons 1600 m.w.e. underground and of the associated electron showers in the air is closely analogous to other experiments performed with ion chambers, counters, and cloud chambers, in which it has been attempted to resolve the core structure of the electron cascades in extensive air showers of a given size.[32–34] The separations of the π^0 mesons (or other, unknown particles) which give rise to the electronic part of the air showers must be similar to the separations of the charged mesons which decay into the μ-mesons seen far underground. In both types of experiments, these separations indicate the angular opening of the cone in which high energy secondaries emerge from the initial or first subsequent nuclear interactions at the origin of the shower; while the total number of electrons or high energy mesons in the air shower allows an estimate of the primary energy.

Up to the present, however, the experiments on the electronic part of air showers have not succeeded in resolving the core structure, and enough is now known to predict no better success in the future. The essential reasons may be outlined as follows:

(a) the electron showers produced by all the different cores overlap, producing a high background density between the cores. The average distance between cores is so small that a core can only be distinguished from this background within a very short radius, inside of which the total number of electrons is small. The linear dimensions vary with the primary energy considered, but this number of electrons is always too small to allow resolution of the cores, for reasons that follow.

(b) Even in simple cascades, there are fluctuations in the local density of electrons.

(c) Parallel to the electronic cascade there is a nucleonic cascade, which can produce energetic π^0 mesons in the lower strata of the atmosphere, resulting in a local core structure superimposed over the original cores. This has been clearly demonstrated by the cloud chamber pictures of Gottlieb[35] and of Branch,[36] which show such local cascades compressed by use of dense plates within the chambers. The net effect when such events occur in the air is to increase both the background density of electrons between the original cores and the fluctuations which are independent of the original core structure.

As a consequence, the only detectable effect of the core structure on the electron density of extensive air showers is to flatten the lateral distribution function near the center of the shower a little, compared with the calculations for a cascade initiated by a single electron or photon. This effect has been reported several times,[34, 37, 38] and is apparently not sensitive enough to the original core structure to provide much information.

The difficulty in detecting a core structure among the electrons arises essentially from attempting to tell the location or direction of a few high energy secondaries from the positions of their numerous scattered descendants of lower energy. A much more sensitive approach would be to measure the positions of the high-energy secondaries themselves. This is approximately equivalent to the measurement, described above, of the decoherence curve of meson showers at a great depth underground.

G. NUCLEAR INTERACTIONS OF μ-MESONS

Evidence for nuclear interactions of μ-mesons is derived by study of the hodoscope records obtained with the apparatus shown in Fig. 1. In this apparatus there were five rows of counters, separated by lead absorbers. Very many events were seen in which secondary particles in addition to a single penetrating particle discharged various counters. In most of these events, the secondaries appeared in only one counter row, hence were of short range and were identifiable as knock-on electrons. From the frequency of these simple events the probability p of a knock-on appearing in one counter row was calculated to be $p=\frac{1}{12}$, with a standard error of about 3 percent.††††

There were also fairly numerous events in which two rows of counters exhibited secondaries. Once again there was evidence of short ranges in most instances; hence we assumed these events, also, were due entirely to knock-on electrons. The frequency of such events involving two counter rows separated by at least 8-in. Pb allowed a second calculation of the probability p. The result agreed with the value $\frac{1}{12}$ given above. For events involving two rows separated by only 2-in. Pb+$\frac{1}{2}$-in. Fe, the rates were higher. This fact permitted a calculation of the probability q for an observed electron shower to penetrate 2-in. Pb+$\frac{1}{2}$-in. Fe. The

[32] R. W. Williams, Phys. Rev. **74**, 1689 (1948).
[33] Cocconi, Tongiorgi, and Greisen, Phys. Rev. **76**, 1020 (1949).
[34] W. E. Hazen, Phys. Rev. **85**, 455 (1952); Hazen, Heineman, and Lennox, Phys. Rev. **86**, 198 (1952).
[35] M. B. Gottlieb, Phys. Rev. **82**, 349 (1951).
[36] G. M. Branch, Phys. Rev. **84**, 147 (1951).
[37] J. M. Blatt, Phys. Rev. **75**, 1584 (1949).
[38] I. D. Campbell and J. R. Prescott, Proc. Phys. Soc. (London) **A65**, 258 (1952).

†††† That this value of p is not higher is due to the large diameter of the counters, 2 inches, which exceeds the normal separation between a meson and its secondary electrons; hence only a fraction of the secondaries were detected as such. For mesons inclined more than 20° relative to the vertical, the selection was even more stringent, since the meson was capable of discharging two adjacent counters. For inclined particles, therefore, the discharge of two counters separated by at least one counter was required in order to tabulate the occurrence of a secondary.

result was $q=\frac{1}{8}$ with a standard error of about 25 percent.

From the values of p and q the frequencies of knock-on showers discharging counters in 3, 4, or 5 counter rows could be calculated and compared with the numbers of showers observed. This was done only for the events occurring while the lead between counter rows D and E was at its maximum thickness, 20 in. The results are in Table VI. There is clearly a difference between calculated and observed rates, which difference represents the sum of penetrating showers created in the ground and events containing coherent mesons created in the atmosphere.

The interpretation of these 109 events as being non-electronic in nature is further supported by examination of the qualitative features of the hodoscope pictures. The counters discharged were frequently separated from each other by several undischarged counters, and the lateral spread of the showers was too large to be accounted for by electrons and photons traversing the absorbers. Indeed, such qualitative examination shows that a small fraction of the events exhibiting secondaries in only one or two counter rows is also due to penetrating showers; but such examination does not permit reliable counting of these penetrating showers because of the peculiar fluctuations that occur in the behavior of electronic showers. Hence the number 109 represents only a lower limit to the number of penetrating showers recorded.

Some of these 109 events, as mentioned above, do not represent local interactions, but pairs or groups of μ-mesons created in the air, which have traversed all the earth above the detector. The number of such events was estimated as 10 from the intercept of the solid curve in Fig. 16. Qualitative examination of the hodoscope pictures yields 20 events that look like pairs of μ-mesons, but about half of this number is expected to represent local interactions anyway, judging from the shape of the graph in Fig. 16 and the discussion on page 160.

By subtracting 10 from the total number of penetrating showers, 109, we find that the number of showers of local origin is (0.8 ± 0.17) percent of the number of mesons. Most of these penetrating showers are observed to be incident on the lead from the rock and salt above the detector instead of being created in the lead. This fact is due to our requirement that 3 or more counter rows be discharged, which does not leave much lead to act as a producing layer.

TABLE VI. Analysis of events in which more than one particle appeared in at least 3 rows of counters separated from each other by layers of lead.

Number of counter rows exhibiting secondaries	3	4	5	Totals
Observed number of events	154	42	16	212
Calculated number due to secondary electrons	96 ± 15	7 ± 2	0.3 ± 0.1	103 ± 15
Difference ascribed to penetrating showers	58 ± 19	35 ± 7	16 ± 4	109 ± 21
Number of pictures considered in tabulation				12,459
Percent containing penetrating events				0.88 ± 0.17
Percent containing penetrating events originating underground				0.80 ± 0.17

The area on which a meson may fall and have its secondary shower detected is larger than the area for detection of a meson without any secondaries. However, mesons may also traverse the counters and have their showers escape detection because the secondaries go off to the side and miss the counters. These effects tend to compensate for each other and no correction is made for them, but they introduce uncertainty in the calculated cross section.

Thus, we equate the ratio $(0.80\pm0.17)\times10^{-2}$ found above to the ratio $R_{\text{sec}}/\lambda_\mu$, where R_{sec} is the mean range in which the observed penetrating showers can be produced and λ_μ is the mean free path for creation of penetrating showers by μ-mesons. The mean range of penetrating showers is known to be several times the interaction mean free path of the secondaries, on account of the nucleonic cascade production; hence we assume $R_{\text{sec}}=300$ g/cm². (This implies a total range of the secondary particles greater than 300 g/cm², because of the lead between the counter trays which had to be traversed.) Thus we obtain for λ_μ and the corresponding cross section σ_μ

$$\lambda_\mu \leqslant (4\pm2)\times10^4 \text{ g/cm}^2$$
$$\sigma_\mu \geqslant (4\pm2)\times10^{-29} \text{ cm}^2 \text{ per nucleon.}$$

As indicated in these results, the expected error is about 50 percent, because of uncertainty in R_{sec} and in the effective area of the detector as well as the statistical error.

The smallness of this cross section and of the capture probability of low energy μ-mesons suggests that the direct coupling with nucleons is negligible, and that the interactions are entirely due to the electromagnetic field. If this is true, it is possible to resolve the electromagnetic field into an equivalent spectrum of photons and express the cross section σ_μ found above in terms of the average cross section σ_ν for nuclear interactions of high energy photons. Following the method of Weizsäcker-Williams, the photon spectrum associated with a meson of energy E is approximately‡‡‡‡

$$f(k)dk=(2/137\pi)\ln(E/k)dk/k,$$

where k is the energy of the photon. The relation between σ_μ and σ_ν is therefore

$$\sigma_\mu=\sigma_\nu\int_{E_m'}^{E} f(k)dk=(\sigma_\nu/137\pi)(\ln E/E_m')^2,$$

where E is the energy of the meson and E_m' is the

‡‡‡‡ This expression is not accurate except for $k\ll E$, and it does not take into account that part of the interaction which is due to the spin of the meson. Since the spin of μ-mesons is $\frac{1}{2}$, that effect is believed to be small.

minimum energy of a photon which can create a penetrating shower like those observed.

We use for E the average energy of mesons after penetrating 1600 m.w.e., which is about 3×10^{11} ev, and we estimate $E_m'=5\times10^{9}$ ev. These values together with the value of σ_μ given above yield

$$\sigma_\nu \gtrsim 1\times10^{-27}\ \text{cm}^2 \text{ per nucleon.}$$

Evidence for penetrating showers created by μ-mesons was also derived in Section F (p. 160) from a consideration of the shape of the decoherence curve of penetrating particles, about half of the pairs at distances less than a meter being ascribed to local showers. Because of the method of selection of these events, the number of local showers included was small, being approximately one per 1000 single mesons detected, or about ten times less than the number of penetrating showers indicated by the data in Table VI. The selection rules required that the counters discharged be consistent with parallel particles traversing all the absorber, and be separated by at least two undischarged counters in each of two trays.

In many of these events the particles did not seem to behave like π-mesons or other strongly interacting particles, which are the usual secondaries of nuclear interactions, but crossed the absorbers in straight lines, after the fashion of μ-mesons, without generating further detectable secondaries. If μ-mesons can be created in interactions with matter, we may have underestimated the mean range of secondary showers, R_{sec}, and hence overestimated the interaction cross section σ_μ. Therefore, we must consider the processes by which μ-mesons or other noninteracting particles might be created in the ground.

In spite of the contradictory evidence of Amaldi *et al.*[14] it is conceivable that the interactions observed by Braddick, Nash, and Wolfendale[13] at 20 m.w.e. underground represent μ-meson production. Even if this is true, however, it does not provide an explanation of the process, nor does it indicate that the same process occurs at greater depths, or that it is caused by mesons.

However, there is a known mechanism by which a small fraction of the secondaries could become μ-mesons, namely, by decay in flight of π-mesons (or κ-mesons) produced directly. The range of π-mesons, determined by nuclear absorption, is on the order of 300 g/cm² or 1.1 meters of rock. The probability of decay before absorption is therefore (since the mean lifetime is 2.6×10^{-8} sec and the mass is 141 Mev/c²)$20/E$, with E in Mev and assuming $E\gg100$ Mev. This is a small probability on the average, but when such an event occurs the μ-mesons are given a longer range than the π-meson would have had. Since $R_\mu\simeq E/2$ g/cm² with E in Mev, the fraction of the mesons emerging from the rock which may be expected to be μ-mesons resulting from $\pi-\mu$ decay is

$$F_\mu \simeq \frac{E}{2\times300}\times\frac{20}{E}\simeq 3 \text{ percent.}$$

Thus in a time during which 100 penetrating showers were observed, each containing on the average, say, three secondary mesons, one would expect about 10 instances when a single secondary μ-meson would appear accompanying the primary μ-meson. This is equal to the number of pairs, recorded at small separations, which were inferred to be local showers in spite of appearing like parallel μ-mesons.

If κ-mesons, which have a shorter lifetime for μ-decay than π-mesons, are sometimes produced in the nuclear interactions, the fraction of the secondaries which decay into μ-mesons in solid matter would be larger than 3 percent; and the pairs appearing like μ-mesons could more easily be accounted for, since one would not have to assume such a high efficiency of detection. Judging from the results of Daniel *et al.*,[15] the energies of the photons responsible for the nuclear interactions are not enough for κ-production to be as frequent as π-production, but the creation of κ-mesons may be frequent enough to multiply the number of secondary μ-mesons emerging from the rock by a factor of two or three.

Decay in solid matter, therefore, is sufficient to account for the observation of a small number of pairs of noninteracting mesons at small spacings. Since this explanation requires only that a few percent of the observed secondaries be μ-mesons and the remainder strongly interacting particles, no appreciable error is indicated in the calculation of σ_μ above. Also, this process does not influence our previous interpretation of the pairs observed at larger distances (2 to 25 meters separation), for reasons that have already been presented and also because the μ-mesons resulting from local interactions are too few by at least two orders of magnitude to account for the coincidences at large separations.

Upper Limit of Cross Section for Neutron Production

In addition to the evidence described above, which essentially yields a lower limit of the nuclear interaction cross section σ_μ, two other kinds of evidence can be presented, which set upper limits to the cross section. These results are particularly of interest because the upper limits are found to be not much greater than the lower limit of σ_μ derived above.

One of the results was obtained with a neutron detecting apparatus, a description of which has already been published.[39] This equipment was first used to study the interactions produced in lead by mesons at small depths (zero to 60 meters) under water. At 1600 m.w.e. in the salt mine, the moderating was accom-

[39] G. Cocconi and V. Cocconi Tongiorgi, Phys. Rev. 84, 29 (1951).

plished with extra blocks of paraffin placed around the equipment. Neutrons produced by interactions in a block of lead, 18 in.×18 in.×4 in. in size, were slowed down in the surrounding paraffin and detected with proportional counters filled with BF_3.

At 1600 m.w.e. the nuclear interactions were so few that not only the single neutrons but also the pairs of neutrons recorded were essentially all due to local radioactive sources, the pairs being accounted for by chance coincidences. In the recording channel in which triple and higher order coincidences were counted, there were 3 events in 335 hours with the lead in place, and 2 events in 320 hours with the lead removed. Therefore the rates of nuclear interactions were judged too low in comparison with the background to permit a positive measurement of the interaction cross section. However, an upper limit can be given by assuming that, at most, all three of the events recorded with the lead in place were due to nuclear interactions produced by cosmic rays in the lead.

We also assume that the multiplicity spectrum of the neutrons of moderate energy produced in the interactions at 1600 m.w.e. is the same as in the interactions occurring at 60 m.w.e. This spectrum determines the efficiency of observing an interaction by detecting three or more of the neutrons produced. By following the methods described in reference 39 one finds

$$(\sigma_\mu)_{\max}=1.2\times10^{-28}\text{ cm}^2\text{ per nucleon in Pb.}$$

This upper limit is three times as great as the value of σ_μ obtained from the penetrating showers seen in the hodoscope, but the difference can be due entirely to the fact that nuclear interactions in which neutrons of moderate energy are produced do not require as much energy as penetrating showers. If the interactions are ascribed, as before, to the virtual photons of the electromagnetic field of the μ-mesons, and the minimum effective photon energy E_m' is set equal to 2×10^8 ev instead of 5×10^9 ev, the neutron experiment yields

$$\sigma_\nu\leqslant1\times10^{-27}\text{ cm}^2\text{ per nucleon,}$$

the same as the lower limit found previously.

Analysis of Intensity *vs* Depth Curve

The second upper limit of σ_μ is derived from the shape of the graph of vertical intensity versus depth underground. The logarithmic derivative, m, of this curve may be written as the product of two factors, one depending on the energy spectrum of the mesons and the other on the rate of energy loss:

$$m=-(h/I)(dI/dh)=[(-E/I)(dI/dE)]\times[(h/E)(dE/dh)],$$

where I is the vertical intensity of mesons penetrating to a depth h underground, and E is the energy required to penetrate to the depth h. The increase of m with depth, shown in Fig. 6, may thus be due in part to each of the following three effects:

(1) Increasing competition between decay and nuclear absorption of the parents of the μ-mesons. If the integral energy spectrum of the parents is a power law with exponent $-\epsilon$, the factor $(-E/I)(dI/dE)$ changes slowly from ϵ to $\epsilon+1$ as the energy increases from a value where nuclear absorption is negligible to one where decay is very improbable compared with nuclear absorption. This change is discussed in the Appendix (Eq. (11)) and illustrated in Fig. 4.

(2) Increase of the exponent ϵ in the energy spectrum of the parents of the μ-mesons. A power law with constant exponent has no experimental or theoretical justification. At low energies (up to 10^{10} ev), measurements of the range and momentum spectra of μ-mesons have shown[40] that if a power law representation of the energy spectrum of the parents is used, the exponent must gradually increase with energy. At high energies, one may estimate the exponent ϵ in the following way. The shape of the energy spectrum of the primary cosmic rays, discussed in Section H and shown in Fig. 19, may be approximated over considerable intervals of energy by a power law, in which the exponent is obtained at any desired primary energy from the slope of the graph in Fig. 19. We assume that the parents of the μ-mesons are created by the primaries with a multiplicity proportional to the $\frac{1}{4}$ power of the primary energy (which is consistent with recent observations of high energy interactions in photographic emulsions,[41] besides being predicted by the Fermi theory). We also assume that the contribution to the spectrum from later generations in the nucleonic cascade is not of enough importance, or of sufficiently different shape, to change the spectrum significantly. This is an expected consequence (at high energies) of the multiple production. Under these assumptions one obtains§§§§

$$\epsilon(E)=[\Gamma(E_p)-\tfrac{1}{4}]/1-\tfrac{1}{4},$$

where Γ is the absolute value of the exponent in the integral primary energy spectrum and E_p is the primary energy mainly responsible for production of the parents of μ-mesons of energy E. E and E_p are related by the $\frac{1}{4}$ power law and the observation (Section E) that μ-mesons of average energy 8×10^{11} ev are associated with primaries of energy 4×10^{13} ev; i.e., $(E_p/10^{10})=11.6(E/10^{10})^{4/3}$. By using the analytic expression for the primary energy spectrum, given in Section H, and combining these relations, with simplifications that are valid for $E\gg10^9$ ev, one obtains

$$\epsilon\simeq1.65+(\tfrac{1}{6})\log_{10}(E/10^{10}).$$

[40] M. Sands, Phys. Rev. **77**, 180 (1950).
[41] M. F. Kaplon and D. M. Ritson, Phys. Rev. **85**, 932 (1952).
§§§§ This result agrees with the one derived from the more complex analysis given by U. Haber-Schaim in Phys. Rev. **84**, 1199 (1951).

If the range spectrum obtained by Sands[40] is converted to an energy spectrum (with E equal to total energy of the mesons rather than kinetic energy), the logarithmic derivative is found to agree well with the above expression at all energies above 10^9 ev. The joining of the low and high energy regions is therefore smooth, and the equation for ϵ may be considered verified between 10^9 and 10^{10} ev. The assumptions and approximations in its derivation are expected to improve with increasing energy.

(3) Increase in the rate of energy loss of μ-mesons with energy, causing an increase in the factor $(h/E)(dE/dh)$. Because of the limitations on the magnitude of the effects (1) and (2), a part of the increase of m with depth must be due to this increase of the energy losses.

The energy losses arise from the following processes:

(a) Ionization and excitation of atoms, and Čerenkov radiation, including in the ionization all secondary electrons up to a fixed energy, η. Because of the polarization effect, the energy loss resulting from these processes saturates for fast mesons at a value $1.94+0.0766\times\ln(\eta/\mu c^2)$ Mev per g/cm² in ground of density 2.65 g/cm³; μc^2 being the rest energy of the meson.[42]

(b) Knock-on electrons of energy above η. Because of the small impact parameters, this process is not affected by polarization, and the energy loss increases logarithmically with the energy: $(dE/dx)_{\text{knock-on}}\simeq 0.0766\times[\ln(E_m'/\eta)-0.8]$ Mev per g/cm², where $E_m'=E^2/(E+\mu^2c^2/2\mu_e)$ is the maximum transferable energy, and μ and μ_e are the masses of meson and electron. The constant $A=\mu^2c^2/2\mu_e$ is 1.13×10^{10} ev.

(c) Bremsstrahlung. The equations of Christy and Kusaka[43] for μ-mesons of spin $\frac{1}{2}$ yield $(dE/dx)_{\text{rad}}\simeq BE\times\ln(E/\mu c^2)$, neglecting screening. The constant B is so small that the radiation loss does not become equal to the ionization loss until an energy around 10^{12} ev. At this energy, the increasing importance of screening should cut off further increase in the logarithmic term. Therefore for the region of importance in our calculations we approximate the logarithm as constant and obtain $(dE/dx)_{\text{rad}}\simeq 1.5\times 10^{-6}E$ per g/cm².

(d) Pair production. Integration of the equations given by Bhabha[44] shows that the energy loss by pair production is similar in magnitude to that by radiation and is also very nearly proportional to the energy: $(dE/dx)_{\text{pairs}}=1.3\times 10^{-6}E$ per g/cm².

(e) Nuclear energy losses. Because of the probable character of these losses, we consider them also to be proportional to the energy, and comparable in magnitude with the radiation losses: $(dE/dx_{\text{nuclear}}=CE$.

Combining these kinds of energy loss, the total rate of energy loss may be written in the form

$$dE/dh=a+k\ln\frac{E_m'}{\mu c^2}+bE$$

with

$$a=1.88\times 10^6 \text{ ev cm}^2/\text{g}$$
$$k=0.0766\times 10^6 \text{ ev cm}^2/\text{g}$$
$$b=(2.8\times 10^{-6}+C)\text{ cm}^2/\text{g}.$$

By using the fact that the logarithm varies slowly with E, we obtain as the approximate solution:

$$E=(a'/b)(e^{bh}-1)$$

with

$$a'=a+k\ln\frac{E^2}{e\mu c^2(E+eA)}.$$

Thus,

$$\frac{h}{E}\frac{dE}{dh}=\frac{bh}{1-e^{-bh}}\left[1-\frac{k}{a'}\frac{E+2eA}{E+eA}\right]^{-1}.$$

At 1600 m.w.e. underground, the logarithmic derivative m was found (Fig. 6 and reference 7) to be 3.5. The energy required to penetrate to this depth is 5 to 6×10^{11} ev, therefore the value of ϵ, from the formula derived above, is 1.95. From Fig. 6 and Section B, the (experimental) change in the exponent due to decay of the parents of the μ-mesons is about 0.65. The derivative $(-E/I)(dI/dE)$ is therefore $1.95+0.65=2.6$. From this and the value of m, one obtains $(h/E)(dE/dh)=1.35$. The uncertainty of ± 0.3 in the effect of the decay was mainly due to the possible error in m; the errors in these two quantities are probably about equal and in the same direction. By taking this into account and assuming an error of ± 0.15 in ϵ, the combined error in $(h/E)(dE/dh)$ is found to be ± 0.10.

The corresponding value of the constant b in the energy loss formula is $(3.5\pm 1.0)10^{-6}$ per g/cm². By subtracting the energy loss due to bremsstrahlung and pair formation, the constant C expressing the nuclear energy loss is found to be

$$\left(0.7\begin{matrix}+1.0\\-0.7\end{matrix}\right)\times 10^{-6}\text{ per g/cm}^2\text{, or}$$

$$(dE/dx)_{\text{nuclear}}<2\times 10^{-6}E\text{ per g/cm}^2.$$

Thus only an upper limit of the nuclear energy loss is obtained. In order to relate the energy loss to a cross section for nuclear interactions, one must make some assumption about the relative probability of losing various fractions of the energy in an interaction. If one represents the interactions as being due to the virtual photons of the electromagnetic field, as was done previously, one may relate the energy loss to the average photonuclear cross section σ_ν:

$$(dE/dx)_{\text{nuclear}}=N_0\sigma_\nu\int^E kf(k)dk=(2N_0\sigma_\nu/137\pi)E,$$

[42] See, e.g., B. Rossi and K. Greisen, Revs. Modern Phys. **13**, 240 (1941).

[43] R. F. Christy and S. Kusaka, Phys. Rev. **59**, 414 (1941).

[44] H. J. Bhabha, Proc. Roy. Soc. (London) **A152**, 559 (1935).

where N_0 is Avogadro's number and k is the photon energy. In this integral the lower limit is unimportant, since it is small compared with E, and may be set equal to zero. The cross section obtained is

$$\sigma_\nu < 0.7\times 10^{-27}\ \text{cm}^2 \text{ per nucleon,}$$

about equal to the upper limit obtained from the neutron experiment and the lower limit obtained from the hodoscope experiment.

It should be pointed out that the energy loss depends sensitively on the upper end of the photon spectrum, $k>10^{11}$ ev, where the representation of the field in terms of virtual photons is inaccurate. The cross section for generating penetrating showers depended more on the photon energy range $10^{10}-10^{11}$ ev, while the meson cross section for producing events in which neutrons of moderate energy are released depended mostly on the photonuclear cross sections below 10^{10} ev. One need not suppose the photonuclear cross section to be constant over such a wide range of energy.

Measurements at Lower Energies

George and Evans[45] by studying stars formed underground in photographic emulsions, inferred a cross section σ_μ equal to about 5×10^{-30} cm^2 per nucleon, at depths from 20 to 60 m.w.e.; while at a depth of 20 meters under water, by detection of neutron showers with the apparatus described above, Cocconi and Tongiorgi[39] found that the cross section σ_μ was about 1×10^{-29} cm^2 per nucleon. These values are an order of magnitude less than the cross section we find at 1600 m.w.e. The average meson energy E at 20 m.w.e. is about 6×10^9 ev. If the interactions are due at both depths to virtual photons of energy $k>E_m\simeq 2\times 10^8$ ev, and σ_ν is constant at energies above E_m, σ_μ is expected to change in proportion to $(\ln E/E_m)^2$ which increases by a factor 4.6 between depths of 20 and 1600 m.w.e. From this viewpoint the measurements are consistent with each other without requiring any change in the photonuclear cross section of photons above 2×10^8 ev.

The cross sections for various types of photonuclear interactions have been measured in numerous recent experiments performed with synchrotrons. A rapid increase in the cross sections for star production, charged meson production, and neutral meson production is observed as the photon energy increases above the meson production threshold, but there is some evidence for levelling-off of the cross section at energies above 300 Mev (or possibly for the existence of a broad maximum at about this energy). Miller[46] has found the cross section for producing stars of two or more prongs in silver to be about 1×10^{-28} cm^2 per nucleon, per quantum of energy around 300 Mev. The cross sections for producing charged π-mesons and for producing neutral π-mesons have also been found to be around 1×10^{-28} cm^2 per nucleon (in light elements). The cross sections for star production and for meson production are not quite additive, since the production of mesons is sometimes associated with the creation of a star. Cross sections are also appreciable for the production of single high energy recoil protons and neutrons,[47] which events must not always be associated with production of a star of two prongs, since the escaping nucleon takes off so much energy.

The photonuclear cross sections have not been measured at all angles in the C-system, and it is not known to what extent the processes listed above are mutually exclusive. It seems likely, however, that the total photonuclear cross section at 300 Mev is between 2 and 3 times 10^{-28} cm^2 per nucleon. At much higher energies, the absorption of a quantum must result in more violent effects, i.e., multiple production of mesons, associated with large stars. If single π-mesons or high energy recoil nucleons were produced, these would have sufficient energy to create a nucleonic cascade within the same nucleus or other nuclei. Thus all of the photonuclear interactions at high energy should lead to penetrating showers and excited nuclei which evaporate neutrons of moderate energy. Therefore one should associate the *total* photonuclear cross section with the cross section for production of penetrating showers at great depths underground.

The value $\sigma_\nu = 2-3\times 10^{-28}$ cm^2 per nucleon at 300 Mev is a few times lower than the photonuclear cross section inferred from the penetrating showers seen at 1600 m.w.e. In view of the uncertainty in these measurements, and the difference in photon energies involved, this is not bad agreement. It supports the conjecture that the interactions observed far underground are due to the Coulomb field of the μ-mesons rather than to other forces. However, since the photonuclear interaction may possibly not represent the entire coupling between a μ-meson and a nucleus, the photonuclear cross section inferred at high energy may be too large. Nevertheless, even if other types of coupling are significant, the electric field remains, and the upper limit of the μ-meson cross section allows one to set an upper limit on the increase of the photonuclear cross section with energy: $\sigma_\nu \leqslant 10^{-27}$ cm^2 per nucleon at energies up to 10^{11} ev, indicating that the cross section does not increase by more than a factor 5 between 3×10^8 and 10^{11} ev.‖‖‖‖

Calculations of the cross section for meson production by electrons or mesons have been made by Sneddon and Touschek[48] and by Strick and Ter Haar[49]; these

[45] E. P. George and J. Evans, Proc. Phys. Soc. (London) **A63**, 1248 (1950); Proc. Phys. Soc. (London) **A64**, 193 (1951). E. P. George, *Progress in Cosmic Ray Physics* (Interscience Publishers, Inc., New York, 1951).

[46] R. D. Miller, Phys. Rev. **82**, 260 (1951).

[47] J. C. Keck, Phys. Rev. **85**, 410 (1952).

‖‖‖‖ We do not take into account the possibility of resonances in the cross section σ_ν. If these exist, the maximum value of σ_ν may have any value, as long as the average value over large regions of the energy spectrum is less than 10^{-27} cm^2.

[48] I. N. Sneddon and B. F. Touschek, Proc. Roy. Soc. (London) **A199**, 352 (1949).

[49] E. Strick and D. Ter Haar, Phys. Rev. **78**, 68 (1950).

theories have been applied to the underground intensity *vs* depth curve by Garelli and Wataghin.[50] These theories account for the meson production essentially as a scattering of charged mesons out of the nucleus by Coulomb interaction with the passing particle. The order of magnitude of the theoretical cross section agrees with all the measurements discussed above, the uncertainty in the constants of the theory being more than enough to cover the range of the experimental discrepancies. However, the theoretical cross sections increase with only the first power of the logarithm of the meson energy. This is equivalent to having a photonuclear cross section that decreases with energy instead of remaining constant or increasing slowly, as our experimental results seem to indicate when compared with either the photonuclear cross section at 300 Mev or the μ-meson interaction cross section at 20 m.w.e. underground.

H. PRIMARY NUMBER-ENERGY SPECTRUM OF THE COSMIC RADIATION

The data and analyses presented above permit estimates of frequency of the primary cosmic rays with energy exceeding two values, the median energy responsible for the single mesons detected at 1600 m.w.e. and the median energy responsible for the pairs. Each energy is estimated in more than one way, one of which involves calculating the primary energy of an air shower having a given number of electrons. The determination of the primary number-energy spectrum differs from previous analyses of air shower data, however, in being largely independent of assumptions about the lateral distribution of the electrons. And in so far as the other means of energy estimation are relied on, the points on the number-energy spectrum are quite independent of air showers. These points therefore add to previous information in establishing the high energy spectrum of cosmic radiation.

The intensity of single mesons, N_1/A_1, can be written as an integral over the differential primary energy spectrum, $I'(E)$:

$$N_1/A_1 = \int_0^\infty I'(E)dE\sum_M MP(E, M)$$

$$= 2\int_{E_{m1}}^\infty I'(E)dE\sum_M MP(E, M) = 2I(E_{m1})\bar{M}_{m1}$$

where E_{m1} is the median primary energy responsible for the mesons, $P(E, M)$ is the probability for a primary of energy E to produce M mesons that penetrate 1600 m.w.e., $I(E_{m1})$ is the number of primaries per cm^2, sec, and sterad having energy above E_{m1}, and $\bar{M}_{m1}$ is the average multiplicity of mesons at 1600 m.w.e. created by the primaries of energy greater than E_{m1}.

The analysis of the air showers associated with the mesons showed that one could successfully approximate $1-P(E, 0)$ by a step function: that is, all the mesons seemed to be associated with air showers above a given size (which presumably is closely correlated with a given primary energy) and nearly all air showers above that size seemed to contain a meson at 1600 m.w.e. This is of course only an approximation, but its success implies that $P(E, 0)$ drops rapidly from one towards zero with increasing energy above the threshold. This behavior can be understood if the production of mesons is multiple, for in that case any interaction capable of producing mesons that can penetrate 1600 m.w.e. will produce a sufficient number of π- or κ-mesons for the probability of one decaying into a μ-meson to be high; and if the interaction is of the violent sort described by Fermi the mesons emitted in the forward direction in the c-system will generally have similar energies.

A more accurate approximation than a step function is to assume that $P(E, 0)$ undergoes the greater part of its variation within the energy range between the threshold and the median energy, being small above the median energy. Then the average of $\Sigma_M MP(E, M)$ over energies greater than E_m will be approximately equal to the average of M over all events in which at least one meson is created that penetrates 1600 m.w.e. This average was found to be 1.20 from the relative frequencies with which single mesons, pairs, and triples were detected.

The absolute intensity N_1/A_1 was found in Section B to be 3.25×10^{-7} cm^{-2} sec^{-1} sterad^{-1} in the vertical direction. From this and the value of $\bar{M}_{m1}$, one finds

$$I(E_{m1}) = 1.35\times10^{-7} \text{ cm}^{-2} \text{ sec}^{-1} \text{ sterad}^{-1}.$$

Because of the uncertainty in $\bar{M}_{m1}$ we attribute to the intensity a 50 percent error.

At energies above the median primary energy E_{m2} responsible for the *pairs* of mesons detected at 1600 m.w.e., the number of mesons created in each event is large, since the median multiplicity M_{m2} is about 10. Therefore, above this energy it is reasonable to assume a unique relation between primary energy and multiplicity of mesons underground. Under this assumption, the intensity $I(E_{m2})$ is equal to the sum of the multiplicity spectrum $F(M)$ from M_{m2} to infinity.

The determination of the spectrum $F(M)$ was discussed in Section F. The shape was determined both by the ratio of triple coincidences to pairs, and by the number of pairs integrated over all counter separations, relative to the number of single mesons recorded; the normalization was determined by the counting rate of single mesons. Corresponding to the exponent $\delta=3.40\pm0.06$ in the spectrum $F(M)$, one finds $I(E_{m2}) = 3.6\times10^{-10}$ cm^{-2} sec^{-1} sterad^{-1} with an uncertainty of 30 percent. Implicit in the derivation is the assumption that the pairs have the same zenith angle distribution as the single mesons detected. Because of this assumption and other sources of error, we estimate the total uncertainty in $I(E_{m2})$ to be about 50 percent.

[50] G. M. Garelli and G. Wataghin, Phys. Rev. **79**, 718 (1950).

The median energy of the primaries responsible for the single mesons observed at 1600 m.w.e. was estimated in Section E, from the sizes of the associated air showers, to be 4×10^{13} ev (within a factor 2). Similarly, the median energy of the primaries responsible for the observed pairs of mesons was judged (in Section F) from the associated showers to be 3×10^{15} ev (within a factor 2 upward and a factor 3 downward).

One may also estimate the two primary energies independently of the air showers, by considering the average energy necessary to produce a single μ-meson or approximately 10 μ-mesons that penetrate 1600 m.w.e. The minimum energy required of μ-mesons in order to penetrate this depth is 6×10^{11} ev and the average energy is close to 10^{12} ev. If the μ-meson is created by $\pi-\mu$ decay, the μ-meson receives on the average only 0.79 of the energy of the π-meson, and only about 10 percent of the π-mesons of this energy undergo decay, the others suffering nuclear absorption. Therefore the energy given to charged π-mesons is about 1.3×10^{13} ev on the average in order to produce a single μ-meson of 10^{12} ev that penetrates 1600 m.w.e. In addition, similar amounts of energy must be given to the nucleons taking part in the interaction, to the neutral π-mesons which give rise to the air showers, and probably to other types of particles (heavy mesons, V-particles, nucleon-antinucleon pairs). This makes the primary energy greater than that of the charged π-mesons by a factor probably between 2 and 4, leading to an estimate of 4×10^{13} ev for the primary energy responsible for the mesons observed singly, in agreement with the estimate based on the air showers.

For the events in which approximately 10 μ-mesons penetrate 1600 m.w.e. underground, one must multiply this energy by a factor greater than 10 because π-mesons of higher energy than 10^{12} ev have smaller probabilities of decaying, and μ-mesons produced in the second generation of the nucleonic cascade would carry a smaller fraction of the total energy than those produced in the first generation. Since the energy of the produced π-mesons increases less rapidly than the primary energy, the factor is certainly less than 100. On the basis of such qualitative arguments alone, one may estimate that the factor is in the range 50 ± 25. In Section F, on the basis of the multiplicity distribution of the meson showers and the density distribution of electrons in air showers, a quantitative relation was derived between multiplicity of mesons and primary energy; and from this relation the above energy ratio was calculated to be 40–50. By accepting $E_{m1}=4\times10^{13}$ ev, one thus obtains $E_{m2}=(2\pm1)\times10^{15}$ ev.

If κ-mesons are the principal parents of the μ-mesons observed underground, the energy estimates made above are not seriously affected. The primary energy must then be divided among κ-mesons as well as $\pi^{\pm}$ and π^0 mesons, and the nucleons, etc. Since the μ-mesons arising from κ-decay seem not to have a unique energy, there are probably three particles emitted, hence the μ-meson receives only about $\frac{1}{3}$ of the energy of the κ, on the average. Furthermore, the data in Sections B and D indicate competition between decay and absorption; therefore either the κ-mesons are not the chief parents of the μ's or else their lifetime is sufficiently long that not all of them decay. (A rather long lifetime is inferred also from the fact that κ-mesons are observed to come to rest in photographic emulsions.) If charged κ-mesons receive $\frac{1}{4}$ of the primary energy and about half of them decay, giving $\frac{1}{3}$ of their energy to a meson of 10^{12} ev, the primary energy responsible for the single mesons underground would be about $2\text{–}3\times10^{13}$ ev, within the uncertainty in the energy estimates based on the air showers and on the $\pi-\mu$ decay scheme.

Alternatively, one may base the estimate of primary energy on the multiplicity of meson production calculated by Fermi.[31] By taking into account the fact that only half of the pions are produced in the forward direction, one finds that the primary energy required to produce μ-mesons of 10^{12} ev is 1 to 2 times 10^{13} ev (depending on the number of other types of particles besides pions that one assumes are produced). It is probable that the multiplicity of produced secondaries is slightly underestimated by the Fermi theory and therefore that the primary energy should be somewhat higher than these values; but it is not likely that the error should be in the other direction. In case the μ-mesons are produced from κ-particles rather than pions, and hence receive a smaller fraction of the energy than in $\pi-\mu$ decay, the primary energy must be greater by a factor of 2 to 3, depending on the decay scheme.

Thus, the average primary energy calculated with the Fermi theory is in fairly good agreement with the estimates based on the sizes of the associated air showers, and on the probability of $\pi-\mu$ decay. However, tht energy given by the Fermi theory is the energy per nucleon, which in the case of heavy primaries is less than the total energy. The agreement is therefore only satisfactory if most of the events are produced by primary protons or possibly alpha-particles, but not by heavy nuclei. This point will be discussed further in the following Section.

Another indication of the value of E_{m2} was obtained from the width of the decoherence curve of the pairs of mesons recorded underground. In Section F, application of the Fermi theory gave 3×10^{14} ev as the effective energy of the particles causing the interactions in which the parents of the mesons are created. Since these interactions must sometimes be produced by secondary particles instead of the primary particle, the median primary energy was inferred to be larger than 3×10^{14} ev by a small factor; hence $E_{m2}\geq10^{15}$ ev. These data provide only a lower limit of the energy, but since the multiplicity M_{m2} is not very large, the expected value of E_{m2} is not far above 10^{15} ev.

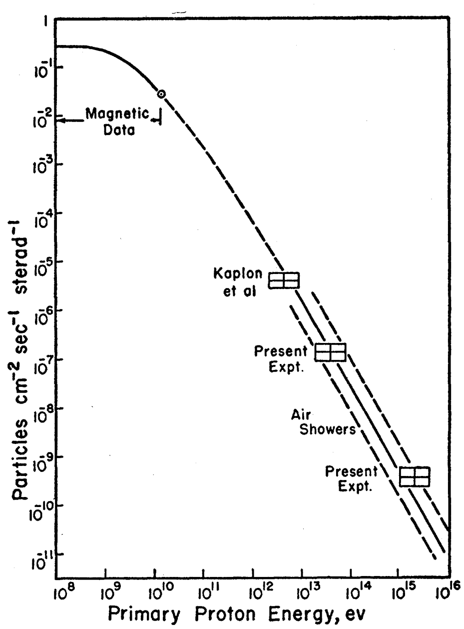

FIG. 19. Number-energy spectrum of the primary cosmic rays. The two dashed lines show the estimated uncertainty in the part of the energy spectrum deduced from the density spectrum of extensive air showers.

By combining all the estimates of energy and intensity outlined above, we conclude that

$$I(4\times10^{13}\text{ ev})=1.35\times10^{-7}\text{ cm}^{-2}\text{ sec}^{-1}\text{ sterad}^{-1}$$
$$I(2\times10^{15}\text{ ev})=3.6\times10^{-10}\text{ cm}^{-2}\text{ sec}^{-1}\text{ sterad}^{-1}$$

with uncertainties of about a factor 1.5 in the intensities and a factor 2 in the energies. In Fig. 19 we have plotted these points together with the data on the primary energy spectrum provided by rocket and balloon experiments at various latitudes,[51,52] and the point at 4.5×10^{12} ev obtained by the Rochester group from large showers observed in an emulsion stack flown at high altitude.[53]

Also shown in Fig. 19, in the region $10^{13}-10^{16}$ ev, is the primary energy spectrum indicated by previous experiments on extensive air showers. The slope of the curve in this region is most directly inferred from experiments on the density distribution of the showers; according to the cascade theory, the exponent in the energy spectrum is approximately equal to the exponent in the density spectrum times the average "age" of the showers detected. The "age" decreases slowly with increasing density, and the exponent in the density spectrum increases slowly, such that the product is nearly constant at approximately 1.8, the slope indicated in the graph. The normalization follows from the calculation outlined on pp. 155–156. By using $\gamma=1.25$ in the density spectrum of showers of small size, one obtains for the frequency of showers with more than 800 electrons, $G(800)$, the value 1.4×10^{-7} cm^{-2} sec^{-1} sterad^{-1}. The corresponding energy was estimated (p. 155) to be 4×10^{13} ev.

The exponent in the part of the energy spectrum derived from air showers agrees with that given by Hilberry[54] in what has become the standard reference on the subject, but the normalization does not. In the older treatments of air showers, the total primary energy was assumed to be in the soft component, the shower was assumed to be initiated by a single photon or electron, a longer value of the radiation length was used, and the assumed lateral distribution was more compressed than that used in the present calculations. All these differences tended to reduce the average primary energy associated with a given density of electrons in the showers. Therefore at a given primary energy the present treatment indicates an intensity 25–30 times as high as that given by Hilberry. The dashed lines in Fig. 19 represent an uncertainty of a factor $\sim$2 in the energy, or $\sim$3 in the intensity at a given energy.

The highest energy point on the number-energy spectrum given by the balloon and rocket data is $I(14\times10^9\text{ ev})\simeq0.028\pm0.002$ cm^{-2} sec^{-1} sterad^{-1}. If one assumes the primary energy spectrum to have a constant exponent between 14 Bev and the energies corresponding to the point obtained by the Rochester group and the two points obtained in the present experiments, these data all yield the same value of the exponent, 1.53, with standard errors ranging from 0.20 to 0.10. Because of the shape of the primary energy spectrum below 14 Bev, however, and the higher exponent indicated by the air shower data, it seems likely that the exponent increases continually with increasing primary energy, reaching a value of 1.7–1.8 in the region 10^{13}–10^{16} ev, and that 1.53 represents only the average value in the region 10^{10}–10^{14} ev. A smooth increase of this sort is indicated by the dashed curve drawn in Fig. 19.

An empirical equation which fits the entire curve drawn in Fig. 19 with imperceptible discrepancies is

$$I=0.3\left(\frac{P}{E+P}\right)^{[1.35+0.02\ln(1+E/P)]}\quad\text{cm}^{-2}\text{ sec}^{-1}\text{ sterad}^{-1}$$

with $P=3.2\times10^9$ ev. The logarithmic derivative of this function, or slope of the curve in Fig. 19, tells the exponent which may be used if the primary energy spectrum is approximated by a simple power law over a limited energy range. This derivative is

$$\Gamma=(-E/I)(dI/dE)=[E/(E+P)]\cdot[1.35+0.04\ln(1+E/P)].$$

Composition and Origin of High Energy Primaries

It has occasionally been suggested in discussions of the origin of cosmic rays[55] that one need not account

[51] J. A. Van Allen and S. F. Singer, Phys. Rev. **78**, 819 (1950).
[52] Winkler, Stix, Dwight, and Sabin, Phys. Rev. **79**, 656 (1950).
[53] Kaplon, Ritson, and Woodruff, Phys. Rev. **85**, 933 (1952).
[54] N. Hilberry, Phys. Rev. **60**, 1 (1941).
[55] R. D. Richtmeyer and E. Teller, Phys. Rev. **75**, 1729 (1949).

for nuclei among the primary cosmic rays with energies per nucleon exceeding 10^{14} ev, because such nuclei have not been observed directly, and the air showers may be due to heavy nuclei or even dust grains, without requiring such great energies per primary nucleon. In fact, the proposal of a solar origin of all the cosmic rays requires the existence of a magnetic field surrounding the solar system, of a strength sufficient to retain all of the cosmic rays. The maximum field strength which could have escaped observation in the investigation of geomagnetism is estimated to be 10^{-4} gauss. In this field, nuclei with $pc/Z=3\times10^{15}$ ev would follow paths with radii of curvature exceeding 10^{17} cm and would rapidly escape to the fields of influence of other stars, even if the field of our solar system extended to a few times 10^{17} cm from the sun (the distances between stars being a few times 10^{18} cm). If the magnetic field is no larger than 10^{-5} gauss, as usually supposed, these particles would wander easily among the stars. On the hypothesis of a solar origin of all the cosmic rays, it is therefore difficult to explain both the high frequency of extensive air showers and the absence of a strong diurnal variation in their intensity, unless the showers are due to nuclei of charge much greater than 1.

Evidence to the contrary is provided by the energy estimates made above. In the case of the primaries of the mesons detected singly, the median Lorentz factor was derived from the average energy of the secondary μ-mesons and from the multiplicity predicted by the Fermi theory; while the total energy was derived from the sizes of associated air showers, and also from a consideration of the $\pi-\mu$ decay probability. In the case of the primaries of the showers of mesons detected underground, the Lorentz factor was obtained from kinematic considerations applied to the decoherence curve, while the total energy was estimated from the air showers, and also from the median multiplicity of mesons in the showers and the dependence of the multiplicity on primary energy. In both cases, the calculated values of the Lorentz factor were more likely to be too low than too high, and yet were only consistent with the estimates of total energy if the primaries are assumed to be protons or alpha-particles, not heavier nuclei.

At a lower total energy, about 5×10^{12} ev, Kaplon, Ritson, and Woodruff[53] found about 90 percent of the events to be caused by particles with no more than single charge, the remaining events being produced by α particles. At still lower energies, where the observations are more numerous, the primaries heavier than α-particles have been shown, not only to constitute a very small fraction of the primaries, but to have an energy spectrum just as steep as that of the protons and α-particles.[56] These observations are consistent with the composition inferred at energies exceeding 10^{15} ev.

Therefore we conclude that the meson showers far underground, and the air showers as well, are produced primarily by protons and α particles; and that the primary cosmic rays include particles with energy orders of magnitude above 10^{14} ev, with an energy spectrum that fits smoothly onto the spectrum at lower energies. The hypothesis of a solar origin of cosmic rays is thus faced with the difficulty of accounting for these particles, and for their lack of a strong diurnal intensity variation.

[56] Kaplon, Peters, Reynolds, and Ritson, Phys. Rev. **85**, 295 (1952).

APPENDIX: EFFECT OF DECAY PROCESSES ON THE ENERGY SPECTRUM OF μ-MESONS

It is the purpose of this appendix to outline the effects on the underground intensity measurements arising from the indirect production of μ-mesons through decay of a short-lived parent. It has been shown that the particles penetrating far underground are ionizing, and since the μ-mesons are the only known ionizing particles having sufficient mass and sufficiently weak nuclear interaction to have ranges exceeding 10^4–10^5 g/cm^2, we assume that μ-mesons and their secondaries are the only particles detected at depths beyond 100 m.w.e. At depths between 20 and 60 m.w.e. direct verification of this assumption has been provided by observations of the particles which reach the end of their range in photographic emulsions.[45]

Until the recent discovery of κ-mesons, the only established mechanism of μ-meson production was the decay of π-mesons, which are known to be produced directly in nuclear interactions. The mean lifetime of π-mesons has been measured, their decay scheme is known, and it has been established that their mean free path for nuclear interaction is close to the geometrical value. Therefore, if one assumes $\pi-\mu$ decay to remain the only significant mechanism of μ-production up to energies beyond 10^{12} ev, and that the π-interaction cross section does not decrease by a large factor at such high energies, one can calculate the expected behavior of the underground intensity down to the greatest depths at which measurements have been made. This is done in the paragraphs below. The predictions can then be compared with experiments to test the validity of the assumptions, as is done in Sections B and D.

The production of μ-mesons by κ-μ decay has by now been observed in a small number of cases, and the observations of Daniel *et al.*[15] indicate that primaries of energy above 50 Bev probably generate κ-mesons about as abundantly as π-mesons. If this is correct, the increasing improbability of $\pi-\mu$ decay with increasing energy should make $\kappa-\mu$ decay an important process for creation of the μ-mesons observed at depths below a few hundred m.w.e. Therefore, following the treatment of the effect of $\pi-\mu$ decay, an estimate is attempted of the changes in the predicted intensity variations if the κ-μ process is substituted for $\pi-\mu$ decay. The lifetime of the κ-meson, the disintegration scheme and the mean free path for nuclear interaction are not well known. A specific model with certain parameters is assumed for the sake of illustration, but the effects are described in a general way so as to facilitate alteration of the parameters. It is even possible that in the future further mechanisms of μ-meson production will be found.

Intensity as a Function of Depth and Zenith Angle

We assume that the production of π-mesons of all energies by other types of particles falls off exponentially with atmospheric depth x along with the line of motion, according to $\exp(-x/\lambda_p)$, λ_p being the absorption mean free path of the producers of π-mesons. We assume the π-mesons retain the same direction as the producing particles, and that the producers are isotropic at the top of the atmosphere. We neglect ionization losses in the atmosphere (for which approximation one may compensate by measuring underground depths from the top of the atmosphere). We consider two absorption processes for the π-mesons: nuclear interaction and $\pi-\mu$ decay. The net fractional loss by nuclear interactions in a thickness dx(g/cm^2) is dx/λ_π, where λ_π is the absorption mean free path of π-mesons, while the fractional loss by decay is $m_\pi c/p'\cdot dx/\rho c\tau_0$, m_π being the mass of the π-meson, p' its momentum, ρ the density of the air, c the velocity of light, and τ_0 the mean lifetime of π-mesons at rest. These assumptions lead to the following equation for the differential π-meson intensity, $\pi(E')$, as a function of x.

$$d\pi(E')/dx = F(E')\exp(-x/\lambda_p)/\lambda_p - \pi(E')[1/\lambda_\pi + m_\pi c/(p'\rho c\tau_0)]. \quad (1)$$

Here $F(E')$ is the differential production spectrum, for which we shall later assume the form $E'^{-(\epsilon+1)}$; and E' is the energy of the π-mesons. Since the energies of interest are high, $p'c=E'$.

We are interested in computing the differential spectrum of μ-mesons arising from decay of the π-mesons. μ-mesons of a given energy E come from π-mesons having a small range of energy, $E<E'<E(m_\pi/m_\mu)^2$, but it is a good approximation to assume that they all arise from π's of the same energy, $E'=E/r$ with $r\simeq m_\mu/m_\pi$.

The density ρ is related to x by $\rho = x\cos\theta/H$ where θ is the zenith angle and $H=RT/Mg$, in which M = molecular weight of air, g = acceleration of gravity, R = gas constant and T = atmospheric temperature. Within the top 250 g/cm^2 of the atmosphere, which is the region of most importance in our calculation, the average values of T and H are practically independent of x. For the present we shall treat H as a constant, giving it the value 6.46×10^5 cm, which corresponds to a weighted average over the atmosphere as discussed later.

By making the substitutions indicated above, and letting $B=m_\pi c^2 rH/c\tau_0$, Eq. (1) becomes

$$d\pi(E')/dx = F(E/r)\exp(-x/\lambda_p)/\lambda_p - \pi(E')[1/\lambda_\pi + B/(Ex\cos\theta)]. \quad (2)$$

B has the dimensions of energy and equals 0.90×10^{11} ev for $\tau_0=2.56\times10^{-8}$ sec, $r=m_\mu/m_\pi=0.76$, $m_\mu c^2=1.07\times10^8$ ev, and $H=6.46\times10^5$ cm.

The solution of Eq. (2) may be written as follows:

$$\begin{aligned}\pi(E',x,\theta) &= [F(E/r)/\lambda_p]e^{-x/\lambda_\pi}x^{-(B/E\cos\theta)} \times \int_0^x e^{-x'/\lambda'}x'^{B/E\cos\theta}dx' \\ &= F(E/r)e^{-x/\lambda_\pi}\frac{x}{\lambda_p}\left\{\frac{1}{1+B/E\cos\theta} - \frac{x/\lambda'}{2+B/E\cos\theta} + \frac{1}{2!}\frac{(x/\lambda')^2}{3+B/E\cos\theta} - \cdots\right\}, \end{aligned} \quad (3)$$

where we have substituted $1/\lambda'=1/\lambda_p - 1/\lambda_\pi$. In case $\lambda_\pi=\lambda_p$, all terms in the brackets except the first one are zero.

From the intensity of π-mesons, the total number of μ's produced by $\pi-\mu$ decay, $M(E)$, is calculable:

$$\begin{aligned} M(E) &= \int_0^{h_0/\cos\theta}\frac{m_\pi c^2}{E\rho c\tau_0}\pi(E')dx \\ &= \frac{1}{r}\frac{B}{E\cos\theta}\int_0^{h_0/\cos\theta}\pi(E')\frac{dx}{x}. \end{aligned} \quad (4)$$

Here h_0 is the total vertical depth of the atmosphere, 1030 g/cm^2, and the upper limit of integration is finite because π-mesons after reaching the earth have a negligible probability to decay before being absorbed. On the assumption that $\lambda_\pi \ll h_0/\cos\theta$ at energies beyond 10^{12} ev, as it is at energies around 10^8 ev, the upper limit of integration will be replaced with infinity. However, if one wishes to consider the results expected for large values of λ_π, one must restore the finite upper limit, which cuts off the increase of $M(E)$ with λ_π.

By substituting Eq. (3) into Eq. (4) and replacing the upper limit of integration with infinity, one obtains

$$M(E) = \frac{F(E/r)}{r}\frac{\lambda_\pi}{\lambda_p}\frac{B}{E\cos\theta}\left\{\frac{1}{1+B/E\cos\theta} - \frac{\lambda_\pi/\lambda'}{2+B/E\cos\theta} + \frac{(\lambda_\pi/\lambda')^2}{3+B/E\cos\theta} - \cdots\right\}. \quad (5)$$

At low energies such that $E\cos\theta \ll B$, this reduces to $M(E)=F(E/r)/r$, expressing the fact that all the produced π-mesons decay. At high energies, such that $E\cos\theta \gg B$, it reduces to $M(E)=[F(E/r)/r]E_0/E\cos\theta$ with

$$E_0 = B[\lambda_\pi/(\lambda_\pi-\lambda_p)]\ln(\lambda_\pi/\lambda_p). \quad (6)$$

A good approximation to the solution (5) at all energies is

$$M(E) = [F(E/r)/r][E_0/(E_0+E\cos\theta)]. \quad (7)$$

This solution is exact in case $\lambda_\pi=\lambda_p$, in which case also $E_0=B$.

One is chiefly interested in the integral spectrum,

$$I(E)=\int_E^\infty M(E^*)dE^*.$$

From Eq. (7), this is equal to the total number of π-mesons produced with energy exceeding E/r, times the average value of the factor $E_0/(E_0+E^*\cos\theta)$ for energies E^* exceeding E. By taking for the production spectrum $F(E')$ the form $(E')^{-(\epsilon+1)}$, a fair approximation to the integral is

$$I(E)\simeq\text{const}\ (r/E)^\epsilon\cdot\frac{E_0}{(\epsilon+1)E\cos\theta+\epsilon E_0}. \tag{8a}$$

The derivative of this function agrees with Eq. (7) for $E\cos\theta\ll E_0$ and $E\cos\theta\gg E_0$ but is too large by about 7 percent for $E\cos\theta$ near E_0, where the error is worst. A better approximation is

$$I(E)=\text{const}\ (r/E)^\epsilon \times\frac{E_0}{(\epsilon+1)E\cos\theta+\epsilon E_0+(E_0E\cos\theta)^{\frac{1}{2}}/(2\epsilon+1)}, \tag{8b}$$

the derivative of which agrees with Eq. (7) within about 2 percent for all values of E.

At low energies Eq. (8b) must be multiplied by a correction factor $D(E)$ to take into account the loss of μ-mesons by decay in the atmosphere. Since we apply the calculations only at depths such that $E>10^{10}$ ev, the factor is always close to one, although not quite negligible. In first approximation it is given by

$$D(E)=\exp\left[-\frac{m_\mu c^2H}{c\tau_\mu E\cos\theta}\frac{\epsilon}{\epsilon+1}\ln\frac{1030}{\lambda_p\cos\theta}\right], \tag{9}$$

where τ_μ is the mean life of μ-mesons and the other symbols have been defined above.

The qualitative features of Eq. (8) and the physical reasons for them are fairly simple. At low energies, since all the produced π-mesons decay, the μ-meson spectrum must be isotropic and have the same form as the production spectrum of π-mesons, $I(E)=\text{const}\ (r/E)^\epsilon$. At high energies, because the decay probability is proportional to 1/E, the spectrum must go as $E^{-(\epsilon+1)}$. The energy E in the neighborhood of which the transition occurs is

$$E=E_0/\cos\theta=(rm_\pi c^2H/c\tau_0\cos\theta) \times(\lambda_\pi/(\lambda_\pi-\lambda_p))\ln(\lambda_\pi/\lambda_p). \tag{10}$$

$E_0/\cos\theta$ is the energy such that the parent π-meson can travel, on the average before decaying, a distance containing λ_π g/cm² of matter. Because of the variation of atmospheric density along the path, this distance is proportional to H, the length in which the atmospheric pressure changes by a factor e, and depends only logarithmically on λ_π. Moreover, even the logarithmic increase is limited by the intervention of the ground if λ_π is not small compared with 1000 g/cm². (The vertical distance to the ground from the 100 millibar level is only 4 times longer than the distance corresponding to a geometric mean free path at 100 millibars.)

The reason for the factor $1/\cos\theta$ is that the π-mesons are produced on the average at higher altitudes by primaries travelling in inclined directions than by primaries travelling vertically. Thus the mean density of the air along the path is proportional to $1/\cos\theta$. Even if λ_π is very long, so that Eq. (8) is not accurate, the dependence of the intensity on $\cos\theta$ remains approximately unchanged, since the average distance from the point of π-meson production to the ground is proportional to $1/\cos\theta$ times a slowly varying logarithmic function of $\cos\theta$.

The significance of the mean free paths λ_p and λ_π, and the assumption that they are constant, may well be questioned. λ_p represents the absorption mean free path of the particles, other than π-mesons, which produce π-mesons; while λ_π represents the absorption mean free path of the π-mesons themselves, and differs from the interaction path because of the generation of π-mesons of lower energy in the interactions of π-mesons. Both λ_p and λ_π therefore depend not only on the cross sections at the energy in question, but on the spectra and cross sections at all energies above the one in question. Since interactions of π-mesons can generate high energy protons, neutrons and other particles as well as π-mesons, and since with increasing energy a larger fraction of the π-mesons undergo such interactions instead of decaying, there is a tendency for both λ_p and λ_π to increase with energy. However, with increasing energy the multiplicity of the produced secondaries goes up, and the increased degeneration of the energy tends to reduce both λ_p and λ_π towards the values of the interaction mean free paths. If the latter are approximately constant, the net changes in λ_π and λ_p are probably small and in the same direction. The effect of such changes would be insignificant, because the exponential nature of the atmosphere makes the results depend on the ratio λ_π/λ_p rather than on the individual values of λ_π and λ_p.

For use in the analysis of the underground intensity, it is desirable to have expressions for the logarithmic derivatives $m=-(h/I)\partial I/\partial h$ and $n=(\cos\theta/I)\ \partial I/\partial\cos\theta$, where h is the vertical depth at which the intensity I is observed, and θ is the zenith angle. Bollinger[7] has shown by a Monte Carlo calculation that in spite of the probabilities of large energy transfers by bremsstrahlung, the fluctuations of the energy losses do not affect the intensity-*vs*-depth relations seriously; hence we assume a unique range-energy relation, $E=E(h/\cos\theta)$

and associate the intensity measured at a depth h in a direction θ with the number of μ-mesons created with energy greater than $E(h/\cos\theta)$. From Eq. (8b) and Eq. (9) one obtains

$$m=\left(\frac{h/\cos\theta}{E}\,\frac{\partial E}{\partial(h/\cos\theta)}\right)\cdot\left(-\frac{E}{I}\,\frac{\partial I}{\partial E}\right)$$

$$=\left(\frac{h/\cos\theta}{E}\,\frac{\partial E}{\partial(h/\cos\theta)}\right)\cdot(\epsilon+\delta-\delta') \tag{11}$$

$$n=m-\delta+\delta'[1+1/\ln(1030/\lambda_p)]\simeq m-\delta+(3/2)\delta' \tag{12}$$

with

$$\delta=1-\frac{\epsilon E_0+(E_0E\cos\theta)^{\frac{1}{2}}/2(2\epsilon+1)}{(\epsilon+1)E\cos\theta+\epsilon E_0+(E_0E\cos\theta)^{\frac{1}{2}}/(2\epsilon+1)}$$

$$\simeq\frac{E\cos\theta}{E\cos\theta+\epsilon E_0/(\epsilon+1)}$$

$\delta'=(1/E\cos\theta)(m_\mu c^2H/c\tau_\mu)(\epsilon/\epsilon+1)\ln(1030/\lambda_p\cos\theta)$.

The term δ describes the change in the spectrum due to the effect of $\pi-\mu$ decay, while δ' gives the correction due to $\mu-e$ decay.

The difference between m and n is plotted in Fig. 4 as a function of depth underground, using for E_0 the value 0.9×10^{11} ev, which corresponds to $\lambda_\pi=\lambda_p$. Values of ϵ, to which the results are insensitive, were obtained by the procedure described in Section G. The effect of $\mu-e$ decay is observed to cancel the effect of $\pi-\mu$ decay at a depth of about 55 m.w.e. underground, where $m=n$.

Dependence of Intensity on Atmospheric Temperature

It is also of interest to infer from these calculations the expected temperature effect of the underground intensity. Qualitatively, at high energies a positive correlation should exist because of the decrease of the density of the air with increasing temperature, which allows more $\pi-\mu$ decay; while at low energies a negative correlation occurs because of the similar dependence of $\mu-e$ decay on density of the air.

For an isothermal atmosphere the expected temperature effect α, defined as the percentage change of intensity per degree change of temperature, can be calculated from Eqs. (8b) and (9), remembering that H and therefore E_0 are proportional to the absolute temperature T. One sees that asymptotically at high energies, the intensity is proportional to T. Therefore α approaches 0.45 percent per degree C, since the temperature near the top of the atmosphere is about 223°K. More generally, one obtains

$$\alpha=(\delta-\delta')/T \tag{13}$$

with δ and δ' as defined under Eq. (12). Figure 8 shows how α would be expected to vary with depth underground, if the atmosphere were isothermal and $\pi-\mu$ decay were the sole means of origin of μ-mesons. The depth at which the positive effect due to $\pi-\mu$ decay would be canceled by the negative effect due to $\mu-e$ decay is about 44 m.w.e. (this is not exact, since our treatment of the $\mu-e$ decay was only approximate).

As discussed in Section D, the actual temperature changes which occur are not uniform throughout the atmosphere, and one cannot approximate the π- or μ-production as occurring entirely at one level. However, the perturbations of intensity caused by variations of temperature are small, and one can treat them in terms of a weighted average or effective temperature of the atmosphere, T_{eff}, which is the temperature of an isothermal atmosphere resulting in the same meson intensity as that produced in an atmosphere having a temperature distribution $T(x)$.

Define $\eta(x)=(T(x)-T_{\text{eff}})/T_{\text{eff}}$ and let $B=B_0(1+\eta)$, where B_0 is the value of B corresponding to $T=T_{\text{eff}}$. One may then solve Eq. (2) to first order in η, treating B as a variable. Let π_0 be the solution, given in Eq. (3), corresponding to $B=B_0$; then the solution for a variable B is $\pi=\pi_0-\pi_1$ where

$$\pi_1(E',x,\theta)=\frac{B_0}{E\cos\theta}e^{-x/\lambda_\pi}\left(\frac{x}{\lambda_\pi}\right)^{B_0/E\cos\theta}\cdot\int_0^x e^{x'/\lambda_\pi}\left(\frac{x'}{\lambda_\pi}\right)^{B_0/E\cos\theta}\frac{\pi_0(x')}{x'}\eta(x')dx'. \tag{14}$$

If $E\cos\theta$ is small compared with B_0, the integrand on the right is large only for x' in the neighborhood of x (i.e., the π-mesons do not travel far); hence $\eta(x')$ can be taken out of the integral and replaced with $\eta(x)$. If $E\cos\theta$ is not small compared with B_0, the correction term π_1 is extremely small compared with π_0. Therefore the second-order error made by removing $\eta(x')$ from the integral is negligible in either case. By making this approximation, one obtains:

$$\pi_1(E',x,\theta)=\eta(x)F(E/r)\frac{x}{\lambda_p}e^{-x/\lambda_\pi}\frac{B_0}{E\cos\theta}\times\left\{\frac{1}{(1+B_0/E\cos\theta)^2}-\frac{x/\lambda'}{(2+B_0/E\cos\theta)^2}+\frac{1}{2!}\frac{(x/\lambda')^2}{(3+B_0/E\cos\theta)^2}-\cdots\right\}. \tag{15}$$

By substituting the expression for π (i.e., $\pi_0-\pi_1$) in Eq. (4), one obtains the μ-meson intensity. Let M_0 be the intensity corresponding to the temperature T_{eff} or the solution π_0; and substitute for B the expression

$B_0(1+\eta)$. Equation (4) then yields, to first order in η:

$$M-M_0=\frac{F(E/r)}{r}\frac{B_0}{E\cos\theta}\int_0^{h_0/\cos\theta}\eta(x)e^{-x/\lambda_\pi}\frac{dx}{\lambda_p}\cdot\left\{\frac{1}{(1+B_0/E\cos\theta)^2}-\frac{2x/\lambda'}{(2+B_0/E\cos\theta)^2}+\frac{3}{2!}\frac{(x/\lambda')^2}{(3+B_0/E\cos\theta)^2}-\cdots\right\}. \quad (16)$$

If $\lambda_\pi=\lambda_p$, all terms beyond the first one within the brackets are zero. If $\lambda_\pi>\lambda_p$, the integrals of the terms beyond the first one are oscillating, as well as progressively decreasing in magnitude. Therefore if $\lambda_\pi>\lambda_p$, the major part of the temperature effect is included in the integral of the first term. If $\lambda_\pi<\lambda_p$, on the other hand, $\lambda'<0$, the series is not oscillating, and the terms beyond the first are not necessarily negligible. In this case a more rapidly converging series is obtainable by replacing $\exp(-x/\lambda_\pi)$ times the series in Eq. (16) with $\exp(-x/\lambda_p)$ times a modified series in powers of x/λ'. The modified series is alternating with progressively decreasing terms, and the first term is the same as in Eq. (16).

Thus, for any relative values of λ_π and λ_p, the major part of the temperature effect is given by the first term on the right side of Eq. (16), provided the value of λ in the exponential factor is set equal to either λ_π or λ_p, whichever is larger.

Up to this point, T_{eff} has been an arbitrary temperature. One may now define T_{eff} in such a way that the integral of the first term in Eq. (16) vanishes, leaving $M\simeq M_0$. This definition of T_{eff} tells how to average the temperatures so that the equation derived for an isothermal atmosphere (Eq. (5)) will include the principal effects of the temperature distribution. The requirement is that $\int_0^{h_0/\cos\theta}\eta(x)e^{-x/\lambda}dx=0$, which implies, from the definition of η,

$$T_{\text{eff}}=\frac{\int_0^{h_0/\cos\theta}T(x)e^{-x/\lambda}dx}{\int_0^{h_0/\cos\theta}e^{-x/\lambda}dx}, \quad (17)$$

in which λ equals λ_π or λ_p, whichever is larger. Since one is usually concerned with the temperature effect of the vertical intensity, the coordinate x may be identified with the atmospheric pressure, and the factor $1/\cos\theta$ in the upper limit of the integral may be dropped. To the best of our knowledge, $\lambda_\pi\simeq\lambda_p\simeq120$ g/cm^2, hence the best known approximation to the weighting factor $e^{-x/\lambda}$ is obtained with $\lambda\simeq120$ g/cm^2.

Estimated Influence of κ-μ Decay

It remains to estimate how the above results would be affected if the significant decay process in the production of μ-mesons were not $\pi-\mu$ decay, but decay of a heavier and shorter-lived meson such as the κ-meson.

In the first place, because of the increased energy available in the center-of-mass system of the decay products, the μ-mesons arising from κ's of a given energy would have a greater spread of energies in the lab system than the μ's resulting from decay of monoenergetic π's. This spread is probably accentuated because, to the best of our knowledge, the κ-meson decays into at least 3 particles and does not produce monoenergetic μ's in the C-system. Therefore our approximation of a constant ratio of energies, $E/E'=r$, may be too crude. However, our knowledge of the decay scheme is too little to permit improvement of this approximation.

Because of the greater mass of the κ-meson, the average value of the ratio r would be smaller than in $\pi-\mu$ decay, where it is about 0.8. If there are 3 decay products, all of small mass compared with m_κ, the average value of r is about $\frac{1}{3}$.

According to the Fermi theory, one should expect roughly equal production of κ- and π-mesons in interactions of very high energy particles. Assuming that equal numbers are produced, with the same energy spectra, proportional to $(E')^{-(\epsilon+1)}dE'$, one can calculate from Eq. (7) the relative number of μ-mesons of a given energy resulting from $\kappa-\mu$ decay and from $\pi-\mu$ decay:

$$M_{\kappa-\mu}/M_{\pi-\mu}=(r_\kappa/r_\pi)^\epsilon(E_{0\kappa}/E_{0\pi})\frac{E_{0\pi}+E\cos\theta}{E_{0\kappa}+E\cos\theta},$$

where

$$E_{0\kappa}=(cHr_\kappa m_\kappa/\tau_\kappa)[\lambda_\kappa/(\lambda_\kappa-\lambda_p)]\ln(\lambda_\kappa/\lambda_p),$$

and a similar expression with subscripts π on the parameters r, m, τ, and λ yields $E_{0\pi}$. Likewise, from Eq. (8) one may obtain the relative number of μ mesons *above* a given energy E, resulting from the two mechanisms. According to the discussion on page 174, however, if λ_κ is comparable with or greater than the thickness of the atmosphere, these expressions are inaccurate; an effective upper limit for $E_{0\kappa}$ is approximately three times the value for $\lambda_\kappa=\lambda_p$.

The effect of $\kappa-\mu$ decay on the slope of the intensity *vs* depth curve, on the difference between the exponents in the dependence of intensity on depth and on zenith angle, and in producing a positive correlation between intensity and temperature, all depend on the quantity δ (see Eqs. (11) to (13)), which is roughly equal to $E/(E+E_0)$. This is not sensitive to changes in E_0 at depths such that $E\gg E_0$, but for smaller depths, δ and all the manifestations of the decay process decrease as E/E_0.

In order to illustrate these relations, the following values of the parameters were assumed, without intending to imply that they are correct:

$r_\kappa = \frac{1}{3}$, $\tau_{0\kappa} = 2\times 10^{-9}$ sec, $m_\kappa = 1200 m_e$, and $\lambda_\kappa = \lambda_p$.¶¶¶¶

Accordingly, $E_{0\kappa} = 2.2\times 10^{12}$ ev. Values of ϵ were obtained by the procedure described in Section G. The expected temperature effect α, and the difference of the exponents m and n in the dependence of intensity on depth and zenith angle, were calculated under the assumption that only κ-mesons are produced at high energies, and also under the more likely assumption that equal numbers of κ- and π-mesons are produced. The results are shown in Figs. 4 and 8, along with the values of $m-n$ and α to be expected if only π-mesons are produced.

In the case of equal production of κ's and π's, it appears that the presence of the κ-mesons probably does not change remarkably the expected values of $m-n$ and α. At depths less than 700 m.w.e., the average energies are too small for competition between decay and nuclear absorption of κ-mesons to be appreciable, but most of the μ-mesons are created by $\pi-\mu$ decay. At depths greater than 700 m.w.e. most of the μ's are created by decay of κ-mesons, but the energies are high enough for the competition between decay and absorption of the κ's to produce effects comparable to those expected from $\pi-\mu$ decay. If the assumed value of $\tau_{0\kappa}$ is too long and λ_κ too short, corrected values of $m-n$ and α would go through a broad maximum at a depth around 500 m.w.e. and decrease slowly thereafter, going through a minimum at a greater depth and then rising again. At depths of 800 and 1600 m.w.e., the values of $m-n$ and α would still be respectively about $\frac{1}{2}$ and $\frac{1}{3}$ of the values expected on the basis of $\pi-\mu$ decay only. Only by assuming that less π-mesons than κ-mesons are produced can the expected values of $m-n$ and α at these depths be made very small.

¶¶¶¶ The reasons for these values are as follows: m_κ is derived from the mass measurement by O'Ceallaigh (see reference 15). For $\tau_{0\kappa}$, a minimum value of about 1×10^{-9} sec is inferred because κ-mesons apparently have a high probability of coming to rest in emulsions before decaying. If κ-mesons are identical with "charged V particles," the observations of decay in the gas of cloud chambers imply that the lifetime is not very long compared with 10^{-9} sec. If this identification is wrong, $\tau_{0\kappa}$ could be much greater than the value assumed, but not much less. $r_\kappa \simeq 1/3$ is inferred from the fact that the charged decay product does not have a unique energy and is light compared with the κ-meson. $\lambda_\kappa \simeq \lambda_p$ seems to be necessary (unless the κ-meson is itself a decay product of a heavier meson) in order to be consistent with the assumption of a high rate of production of κ-particles.

Erratum: The Diamagnetism of Ions

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THE heading for Table VIII should have read as follows:

Table VIII. Ionic diamagnetic susceptibilities.

	Free	Experimental Dissolved			Crystal	Theoretical			
	1	2	3	4	5	6	7	8	9
F^-	10.5	—	12.1	—	9.4	8.1	16.9	8.1	7.1
....	.....	.....	.....	.....	......	.....	.....	.....	.....